9. Evaluate $\int_{|z|<3} \dfrac{dz}{(z-4)z}$.

10. State the Poisson Integral Formula.

11. Find the radius of convergence of the power series, $\sum_{n=1}^{\infty} \dfrac{z^n}{n^{\alpha}}, \alpha \in R$.

12. Define removable singularity and explain it with an example.

13. Define a Maclaurin series. Write the Maclaurin series expansion of $\sin z$.

14. Find the residues at the singular points of $f(z) = \dfrac{8-z}{z(z-4)}$.

15. State Jordan's lemma.

PART – C

(Answer **any four** questions. **Each** question carries **4** marks.)   (4×4=16)

16. Construct an analytic function whose real part is $u(x, y) = 3x^2y + 2x^2 - y^3 - 2y^2$.

17. Determine whether the functions
    i) $f(z) = x^2 - y^2 - 2ixy$ and
    ii) $f(z) = \dfrac{1}{1+|z|^2}$ are entire.

18. State and prove Morera's theorem.

19. Find $\int_C \bar{z}\, dz$ in the counter clockwise direction, where C is triangle with vertices
    0, 1 + i and −2.

20. Find the power series expansion in the neighbourhood of zero, of
    $f(z) = \dfrac{1}{z^2 - 5z + 6}$.

21. Compute $\int_0^{\pi} \dfrac{d\theta}{2-\cos\theta}$.

PART – D

(Answer **any two** questions. **Each** question carries **6** marks.)   (2×6=12)

22. Prove that $u = e^{-x}(x\sin y - y\cos y)$ is harmonic. Find the imaginary part of $f(z)$ with u as real part.

23. Prove the Cauchy's integral formula for n^{th} order derivative.

24. Find the Laurent's series expansion of the function
    $f(z) = \dfrac{1}{z^2 - 3z + 2}$
    valid in
    i) $|z| < 1$
    ii) $1 < |z| < 2$ and
    iii) $|z| > 2$.

25. Using residue theorem, evaluate $\int_{|z|=3} \dfrac{1}{z^2(z-2)(z+1)}\, dz$.

PART – A

(Answer **all** questions. **Each** question carries **1** mark.)   (6×1=6)

1. Define an analytic function.

2. Write down the polar form of Cauchy-Riemann equations.

3. State Cauchy's theorem.

4. Define zero of a complex function.

5. State Cauchy's residue theorem.

6. Determine the poles and their orders of the function $f(z) = \dfrac{1}{z(z-2)^3}$.
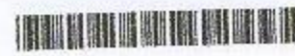
PART – B

(Answer **any seven** questions. **Each** question carries **2** marks.)   (7×2=14)

7. Show that if f(z) is an analytic function, then it satisfies the Cauchy-Riemann equations.

8. Verify the Cauchy-Riemann equations if $f(z) = e^{i(x-iy)}$.