13. Determine the order of zero of $(z^2-1)^2(e^{z^2}-1)$.
14. Define singularity and its different types with examples.
15. State Jordan's lemma.

PART – C

Essay. Answer **any 4** questions. **Each** question carries **4** marks.

16. Derive Cauchy-Reimann equations in polar co-ordinates.
17. Integrate $f(z)=\frac{z^2}{(z-i)^2}$ counter clockwise around the circle $|z|=2$.
18. Show that if a function $f = u + iv$ is analytic in a domain D iff v is the harmonic conjugate of u.
19. Without evaluating the integral, show that $\left|\int_c \frac{dz}{z^2-1}\right| \le \frac{\pi}{3}$, where C is a circle $|z| = 2$ from $z = 2$ to $z = 2i$.
20. State Taylor's formula. Find Taylor series expansion of $f(z)=\frac{1}{(1+z^2)}$ at $z = 0$.
21. Find the order of the pole of the function $f(z)=\frac{1-e^{2z}}{z^4}$ and find its residue.

PART – D

Long essay. Answer **any 2** questions. **Each** question carries **6** marks.

22. Let f be continuous on a domain D and $\int_c f(z)dz = 0$ for every closed contour C in D, then show that f is analytic throughout D.
23. State and prove Cauchy integral formula.
24. Show that the two power series $\sum_{n=0}^{\infty} a_n z^n$ and $\sum_{n=1}^{\infty} a_n z^{n-1}$ have same radius of convergence.
25. State Cauchy's residue theorem. Use it to evaluate the integral $\int_c \frac{5z-2}{z(z-1)} dz$ where C is the circle $|z| = 2$.

K22U 0438

Reg. No. : ....

Name : ....

VI Semester B.Sc. Degree (CBCSS – OBE – Regular) Examination, April 2022
(2019 Admission)
CORE COURSE IN STATISTICS
6B11STA : Mathematical Methods for Statistics – III

Time : 3 Hours Max. Marks : 48

PART – A

Short answer. Answer **all** questions. **Each** question carries **1** mark.

1. How would you test for analyticity ?
2. Define Harmonic function.
3. Show that $f(z) = z^2 + 2$ is entire.
4. Evaluate $\int_i^{2+i} z\,dz$.
5. Determine what kinds of singularities have the function $\frac{1-e^z}{1+e^z}$ at $z = \infty$.
6. What is a pole ?

PART – B

Short essay. Answer **any 7** questions. **Each** question carries **2** marks.

7. Show that the function $f(z) = e^{\bar{z}}$ is not analytic everywhere.
8. Illustrate Cauchy's theorem with an example.
9. Evaluate $\int 4z - 3$ where C is a straight line segment from i to 1 + i.
10. Show that $\int \left(e^{\frac{1}{z^2}}\right) dz$ is 0.
11. State and prove Liouville's theorem.
12. Determine the centre and the radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{(2n)!}{(n!)^2}(z-3i)^n$.