K22U 0151

Reg. No. : .....

Name : .....

VI Semester B.Sc. Degree (CBCSS – Supple./Improv.)
Examination, April 2022
(2016-2018 Admissions)
CORE COURSE IN STATISTICS
6B10STA : Mathematical Analysis – II

Time : 3 Hours Max. Marks : 48

***Instruction** : **Use** of Calculators and Statistical tables are **permitted**.*

PART – A
Short Answer

(Answer **all** the questions) **(6×1=6)**

1. Define Riemann integral of a function.
2. Show that the constant function f(x) = k, defined on [a,b] is integrable.
3. State the fundamental theorem of integral calculus.
4. Define linear independence of vectors.
5. State the Taylor's theorem of a function of two variables.
6. What do you mean by an improper integral ?

PART – B
Short Essay

(Answer **any seven** questions) **(7×2=14)**

7. Prove that the refinement of a partition decreases the Riemann upper sum.
8. State and prove a sufficient condition for the integrability of a function.

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9. Define primitive of a function and explain it with an example.
10. Discuss the integrability of the function f(x), where $f(x) = \begin{cases} 1 & \text{when } x > 0 \\ 0 & \text{when } x \le 0 \end{cases}$.
11. Find the dot product of the vectors, $V_1 = (2, 1, 1)$ and $V_2 = (1, -1, 3)$. Are they orthogonal vectors ? Why ?
12. Create orthonormal system of vectors for R^3, based on the vectors $U_1 = (1, 1, 1)$, $U_2 = (1, 0, -1)$, $U_3 = (1, -2, 1)$.
13. Find $\lim_{(x,y)\to(0,0)} \frac{x^2y^2}{x^2+y^2}$.
14. Find the stationary points of the function $x^2 + y^2 + x + y + xy$.
15. Test the convergence of $\int_0^1 \frac{dx}{\sqrt{1-x^2}}$.

PART – C
Essay

(Answer **any four** questions.) **(4×4=16)**

16. If f is integrable on [a, b], discuss the integrability of f^2 in [a,b].
17. Show that every continuous function is integrable.
18. Define eigen values and eigen vectors of a square matrix. Also show that the eigen values of a null matrix are zeros.
19. Show that linear independence in a system of vectors is not altered by non-zero scalar multiplication of the vectors.
20. Show that $z = x\cos(y/x) + \tan(y/x)$ satisfies $x^2z_{xx} + 2xyz_{xy} + y^2z_{yy} = 0$ except at points for which x = 0.
21. Show that an absolutely convergent integral is convergent.

PART – D
Long Essay

(Answer **any two** questions.) **(2×6=12)**

22. i) State and prove the first mean value theorem.

    ii) If f is a continuous function, such that $f \ge 0$, and $\int_a^b f\,dx = 0$, prove that f(x) = 0 for all x in [a,b].

23. Determine the eigen values and eigen vectors of the matrix $\begin{bmatrix} 1 & 2 \\ 3 & -4 \end{bmatrix}$.
24. Examine the function $x^3 + y^3 - 3x - 12y + 20$ for maximum and minimum.
25. Define Beta integral and discuss its convergence.

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