K16U 1343

Reg. No. : ....

Name : ....

II Semester B.Sc. Hon's (Mathematics) Degree (Reg./Supple./Improv.) Examination, May 2016

BHM 204 : DISCRETE MATHEMATICS

Time : 3 Hours Max. Marks : 80

Answer **all** the **10** questions. **(10×1=10)**

1. Define a culn2 graph. Give an example.
2. Define the join G + H of two vertex-disjoint graph G and H. Give an example.
3. Define a tournament. Give an example.
4. Define a Unicycle graph. Give an example.
5. State Whitney's theorem.
6. Define a Hamiltonian graph. Give an example.
7. Using Venn diagram Prove that $N(\bar{c}_1 \bar{c}_2) = N - [N(c_1) + N(c_2)] + N(c_1 c_2)$.
8. Write the generating functions for the sequence 1, 1, 1, ...1, 0, 0,...... 0 where the first (n + 1) terms are 1.
9. Prove that Pd(6) = 4.
10. Prove that $\frac{e^x + e^{-x}}{2} = x + \frac{x^3}{3} + \frac{x^5}{5} + ...$

Answer **any 10** short answer questions out of **14**. **(10×3=30)**

11. State and prove Handshaking lemma.
12. For every nontrivial connected graph G. Prove that Vad (G) $\leq$ diam (G) $\leq$ 2 Vad (G).
13. Consider the graphs $G = P_3$ and $H = P_3$. Prove the following :

    i) $G \cup H = 2P_3$

    ii) $G + H = P_3 + P_3$

    iii) $G \times H = P_3 + P_3$

P.T.O.

14. Prove that a graph G is a free if and only if every two vertices of G are connected by a Unique path.

15. Prove that for every positive integer n λ (Kx) = n – 1.

16. Prove that the graph B(2, 4) is Exterian

17. If G is Hamiltonian then prove that $K(G\text{-}S) \leq |S|$ for every non-empty proper subset S of V(G).

18. Prove that every r-regular bipartite graph ($r \geq 1$) has a perfect matching.

19. In how many ways can the 26 letters of the alphabet be arranged so that none of the patterns car, dog, pun or byte occurs ?

20. Define Euler's Phi function and hence determine ϕ (23100).

21. Approximate $\frac{1}{e}$ correct to five decimal places.

22. In how many ways can a police captain distribute 24 rifle shells to four patrol men so that each patrolmen gets at least three, but not more than 8 shells ?

23. Using generating function determine how many 4 elements subset of S = { 1, 2, 3,..... 15} contain no consecutive integers.

24. Prove that $(1+3x)^{-1/3}$ is the generating sequence for 1, –1, (–1) (–4)/2!, (–1) (–4) (–7)/3!........... (–1) (–4) (–7)...... (–3r + 2)/r!..........

Answer **any 6** out of 9. **(6×5=30)**

25. Prove that a nontrivial graph G is bipartite if and only if G contains no odd cycle.

26. For every graph G prove that $K(G) \leq \lambda(G) \leq \delta(G)$.

27. Let G be a graph of order $n \geq 3$. If $\deg u + \deg v \geq n$ for each pair u, v of non adjacent vertices of G, then prove that G is Hamiltonian.

28. Prove that every bridgeless cubic graph contains a perfect matching.

29. For every positive integer K, prove that the complete graph K_{2K+1} is Hamiltonian factorable.

30. Determine the number of positive integers n where $1 \leq n \leq 100$ and n is not divisible by 2, 3 or 5.

31. While at the race track, Ralph bets on each of the ten horses in a race to come in according to how they are favoured. In how many ways can they reach the finish line so that he loses all of his bets ?

32. Determine the coefficient of x^8 in $\frac{1}{(x-3)(x-2)^2}$.

33. A company hires 11 new employees. Each of these employees is to be assigned to one of four subdivisions with each sudivisions getting atleast one new employee. In how many ways can these assignments be made ?

Answer **any one** essay questions. **(1×10=10)**

34. State and prove the principle of inclusion and Exclusion. Discuss its generalization.

35. Let G be a bipartite graph with partite sets U and W, where $|U| \leq |W|$. Then prove that U can be matched to a subset of W if and only if Hall's condition is satisfied.

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