If  $T=L^2$  ([-  $\pi$ ,  $\pi$ )) is a bounded, translation-invariant lines undefinition, then show that there exists  $L_m \in \mathbb{C}$  such that T ( $e^{(m)}$ ) =  $L_m e^{(m)}$  for each  $m \in \mathbb{Z}$ 

Ht - vinU

To before approximate identity. Suppose  $(*L^1(R))$  and  $(g_i)_{i=0}$  is an approximate identity. Then allow mall for every Unbelique point s of f. Top g s f(s) = f(s)

(8) Define Fourier transform and inverse Fourier transform on R. Suppose  $f = L^1(R)$  and h(R), then show that  $\frac{1}{2\pi} \left[ \frac{1}{2} (R)^{-1} (R) \right]$  are on R. Lee tric to establish

the uniqueness of Foundi tensions:

9: Suppose tel. (R). (L) (R) as n exquence of functions such that f<sub>iv</sub> f<sub>iv</sub> ∈ L'(R) for each n and f<sub>iv</sub> − f in L\* (R) as n − ∞. Show that (b) converges to a unique

FEL<sup>®</sup>(P) Also show were the L<sup>\*</sup>(P) Ten L<sup>\*</sup>(P) then F L<sup>\*</sup>

Or 11. g of 2000, one on that U. 25 = 20.5, gr. and U. - 2015 II had given on the fitting as new, usen proved in a section of the contract of



K21P 0242

Reg. No. : ......

Name : .....

IV Semester M.Sc. Degree (CBSS – Reg./Suppl. (Including Mercy Chance)/Imp.) Examination, April 2021

(2017 Admission Onwards)

MATHEMATICS

MAT 4E02 : Fourier and Wavelet Analysis

Time: 3 Hours Max. Marks: 80

(2.5) Suppose M is a natural rule = TRACEM and zelf (2.1). Deline zelf (2.1)

Answer any four questions from this Part. Each carries 4 marks.

- 1. Define the conjugate reflection of  $\omega \in l^2(Z_N)$ . For any  $z, \omega \in l^2(Z_N)$  and  $k \in Z$ , prove that  $z * \widetilde{\omega}(k) = \langle z, R_k \omega \rangle$ .
- 2. Explain downsampling operator and upsampling operator.
- 3. If N = 2M for some natural number M,  $z \in l^2(Z_N)$  and  $\omega \in l^2(Z_{N/2})$ , then prove that  $D(z)*\omega = D(z*U(\omega))$ .
- 4. If  $z \in l^2(Z)$  and  $\omega \in l^1(Z)$ , show that  $z * \omega \in l^2(Z)$  and  $||z * \omega|| \le ||\omega||_1 ||z||$ .
- 5. Define translation-invariant linear transformation on  $l^2(Z)$ . If  $T: l^2(Z) \to l^2(Z)$  is bounded and translation-invariant, then show that T(z) = b\*z for all  $z \in l^2(Z)$ , where  $b = T(\delta)$ .
- 6. If  $z \in l^2(Z)$ , show that  $(z^*)^*(\theta) = z^*(\theta + \pi)$ .
- 7. If f,  $g \in L^1(R)$ , show that  $f * g \in L^1(R)$  and  $||f * g||_1 \le ||f||_1 ||g||_1$ .
- 8. If f,  $g \in L^1(R)$  and if f,  $\hat{g} \in L^1(R)$ , prove that  $\langle \hat{f}, \hat{g} \rangle = 2\pi \langle f, g \rangle$ . (4×4=16)

9.3 (II) 5(0) 5 = (II) You'd Half syon; (X) Loos bits (X) 10 E II (U P.T.O.







## PART - B

-2-

Answer any four questions from this Part, without omitting any Unit. Each question carries 16 marks.

## Unit - I

- 9. a) Let  $w \in l^2(Z_N)$ . Then show that  $\{R_k w\}_{k=0}^{N-1}$  is an orthonormal basis for  $l^2(Z_N)$  if and only if  $|\widehat{w}(n)| = 1$  for all  $n \in Z_N$ .
  - b) Suppose M is a natural number, N = 2M and  $z \in l^2(Z_N)$ . Define  $z * \in l^2(Z_N)$  by  $z * (n) = (-1)^n z(n)$  for all n. Then show that  $(z^*)^n(n) = \hat{z}(n + M)$  for all n.
  - 10. Suppose M is a natural number and N = 2M. If u,  $v \in l^2(Z_N)$ , show that  $\left\{R_{2k} v\right\}_{k=0}^{M-1} \cup \left\{R_{2k} u\right\}_{k=0}^{M-1}$  is an orthonormal basis for  $l^2(Z_N)$  if and only if the system matrix A(n) of u, v is unitary for each n = 0, 1, ..., M -1.
  - 11. Suppose M is a natural number, N = 2M and u, v, s, t  $\in l^2(Z_N)$ . Show that  $\tilde{t}*U(D(z*\tilde{v}))+\tilde{s}*U(D(z*\tilde{u}))=z$  for all  $z\in l^2(Z_N)$  if and only if  $A(n)\begin{bmatrix}\hat{s}(n)\\\hat{t}(n)\end{bmatrix}=\begin{bmatrix}\sqrt{2}\\0\end{bmatrix}$  for each n = 0, 1, ..., N 1, where A(n) is the system matrix of u, v.
  - If 2<sup>p</sup> | N, explain the construction of a p<sup>th</sup> stage wavelet basis for t<sup>2</sup> (Z<sub>N</sub>) from a given p<sup>th</sup> stage wavelet filter sequence.

## Unit - II

- 13. a) If  $\{a_j\}_{j\in Z}$  is an orthonormal set in a Hilbert space H and if  $f\in H$ , show that the sequence  $\left\{\left\langle f,\ a_j\right\rangle\right\}_{i\in Z}\in\ell^2(Z)$ .
  - b) Show that an orthonormal set  $\{a_j\}_{j\in Z}$  in a Hilbert space H is a complete orthonormal set if an only if  $f=\sum_{i\in Z}\left\langle f,a_i\right\rangle a_i$  for all f in H.
- 14. a) Suppose  $f \in L^1([-\pi, \pi))$  and  $(f, e^{in\theta}) = 0$  for all  $n \in Z$ . Show that  $f(\theta) = 0$  a.e.
  - b) If  $z \in l^2(Z)$  and  $\omega \in l^1(Z)$ , prove that  $(z*\omega)^*(\theta) = \hat{z}(\theta)\hat{\omega}(\theta)$  a.e.





K21P 0242

- 15. If  $T:L^2([-\pi,\pi))\to L^2([-\pi,\pi))$  is a bounded, translation-invarient linear transformation, then show that there exists  $\lambda_m\in C$  such that  $T(e^{im\theta})=\lambda_m e^{im\theta}$  for each  $m\in Z$ .
- 16. Suppose that  $u, v \in l^1(Z)$ . Show that  $B = \{R_{2k}v\}_{k \in Z} \cup \{R_{2k}u\}_{k \in Z}$  is a complete orthonormal set in  $l^2(Z)$  if and only if the system matrix  $A(\theta)$  is unitary for all  $\theta \in [0, \pi)$ .

## Unit - III

- 17. Define approximate identity. Suppose  $f \in L^1(R)$  and  $\{g_t\}_{t>0}$  is an approximate identity. Then show that for every Lebesgue point x of f,  $\lim_{t\to 0^+} g_t * f(x) = f(x)$ .
- 18. Define Fourier transform and inverse Fourier transform on R. Suppose  $f \in L^1(R)$  and  $\hat{f} \in L^1(R)$ , then show that  $\frac{1}{2\pi} \int_{R} \hat{f}(\xi) e^{ix\xi} \, d\xi = f(x)$  a.e. on R. Use this to establish the uniqueness of Fourier transform.
- 19. Suppose  $f \in L^2(R)$ ,  $\{f_n\}_{n=1}^m$  is a sequence of functions such that  $f_n$ ,  $\hat{f}_n \in L^1(R)$  for each n and  $f_n \to f$  in  $L^2(R)$  as  $n \to \infty$ . Show that  $\{\hat{f}_n\}_{n=1}^m$  converges to a unique  $F \in L^2(R)$ . Also, show that if  $f \in L^1(R) \cap L^2(R)$ , then  $F = \hat{f}$ .
- 20. If  $f, g \in L^2(R)$ , prove that  $\langle \hat{f}, \hat{g} \rangle = 2\pi \langle f, g \rangle$  and  $\|\hat{f}\| = \sqrt{2\pi} \|f\|$ . If  $f \in L^2(R)$  and if  $\{f_n\}_{n=1}^{\infty}$  is a sequence of  $L^2$  functions such that  $f_n \to f$  in  $L^2(R)$  as  $n \to \infty$ , then prove that  $\hat{f}_n \to \hat{f}$  in  $L^2(R)$  as  $n \to \infty$ . (4×16=64)