K17P 0409

Reg. No. : ……………………………

Name : ……………………………

Fourth Semester M.Sc. Degree (Reg./Supple./Imp.) Examination, March 2017
(2014 Admission Onwards)
MATHEMATICS
MAT4E05 : Fourier and Wavelet Analysis

Time : 3 Hours Max. Marks : 60

Instructions : 1) *Notations are as in* ***prescribed*** *text book.*
2) *Answer* ***any four*** *questions from Part – **A**.* ***Each*** *question carries* ***3*** *marks.*
3) *Answer* ***any four*** *questions from Part – **B** without omitting* ***any*** *Unit.* ***Each*** *question carries* ***12*** *marks.*

PART – A

1. Define conjugate reflection of $w \in l^2(\mathbb{Z}_N)$ show that $(\tilde{w})\wedge(n) = \overline{w\wedge(n)}$.

2. Explain how one can generate a p^{th} stage wavelet basis. (Make suitable assumptions)

3. Prove that $l^1(\mathbb{Z})$ is closed w.r.t. convolution.

4. Does there exist a frequency localized orthonormal basis for $l^2(\mathbb{Z})$ consisting of translates of a vector in $l^1(\mathbb{Z})$?

5. For $f \in L^1(\mathbb{R})$, prove that $(R_y f)\wedge(\xi) = e^{-iy\xi} f^\wedge(\xi)$ a.e.

6. Prove or disprove. There is an identity for convolution in $L^2(\mathbb{R})$. **(4×3=12)**

PART – B

Unit – 1

7. a) Show that $u = \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, 0, 0, 0, 0\right)$ is a father wavelet.

b) Find out a mother wavelet v so that u and v generate a first stage wavelet basis.

c) Write down the signal $z = (4, 8, -8, 4, 8, 16, 4, -4)$ in terms of the above first stage wavelet basis.

P.T.O.

8. Let $N = 2^n$, $1 \le p \le n$ and $z \in l^2(\mathbb{Z}_N)$. .

a) Define a p^{th} stage wavelet filter sequence.

b) Briefly describe how to obtain the output of the analysis phase of the p^{th} stage wavelet filter bank corresponding to the input $z \in l^2(\mathbb{Z}_N)$.

c) Show that the analysis phase of the corresponding p^{th} stage wavelet filter can be computed in no more than $4N + N\log_2 N$ complex multiplications.

9. a) Define up sampling and down sampling operators. If N is even, show that

$$(Dz)^{\wedge}(n) = \frac{1}{2}\left[\hat{z}(n) + \hat{z}\left(n + \frac{N}{2}\right)\right], \text{ for } z \in l^2(\mathbb{Z}_N).$$

b) For $z \in l^2(\mathbb{Z}_N)$, prove that $(Uz)^{\wedge}(n) = \hat{z}(n)$ for all n.

Unit – 2

10. a) Define the spaces $L^1[-\pi, \pi)$ and $L^2[-\pi, \pi)$. Give a norm on each and verify the conditions.

b) Show that $L^2[-\pi, \pi)$ is strictly contained in $L^1[-\pi, \pi)$.

c) Prove that point wise convergence doesn't imply norm convergence in $L^2[-\pi, \pi)$.

11. a) Show that a bounded translation invariant linear transformation $T: L^2[-\pi, \pi) \to L^2[-\pi, \pi)$ is diagonalised by the complex trigonometric system $\{e^{in\theta}\}_{n \in \mathbb{Z}}$.

b) Show that the result in part(a) is not true with real trigonometric system $\{1\} \cup \{\sqrt{2}\cos n\theta\}_{n \in \mathbb{Z}} \cup \{\sqrt{2}\sin n\theta\}_{n \in \mathbb{Z}}$ by considering the function $T_j: L^2[-\pi, \pi) \to L^2[-\pi, \pi)$ defined by $T_j(f) = \langle f, e^{ij\theta} \rangle e^{ij\theta}$.

12. a) Suppose $u \in l^1(\mathbb{Z})$ is such that $\{R_{2k}u\}_{k \in \mathbb{Z}}$ is an orthonormal set in $l^2(\mathbb{Z})$. Show that this can be extended to a complete orthonormal system (i.e., first stage wavelet system) for $l^2(\mathbb{Z})$.

b) Let $p \in \mathbb{N}$. For $l = 1, 2, ..., p$ assume that $u_l, v_l \in l^1(\mathbb{Z})$ are such that the corresponding system matrix $A_l(\theta)$ is unitary for all $\theta \in [0, \pi)$. Suitably defining functions, explain the construction of a p^{th} stage wavelet system for $l^2(\mathbb{Z})$.

Unit – 3

13. a) Define convolution of two functions $f, g: \mathbb{R} \to \mathbb{C}$ by making suitable assumptions. Establish that it is commutative.

b) For $f, g \in L^1(\mathbb{R})$, show that $f * g \in L^1(\mathbb{R})$ and that $(f * g)^{\wedge} = \hat{f}\hat{g}$.

14. a) State and prove Fourier inversion theorem on $L^1(\mathbb{R})$.

b) Show that if two $L^1(\mathbb{R})$ functions have same Fourier transforms a.e., the functions are equal a.e.

15. a) For $f, g \in L^2(\mathbb{R})$, prove the relation $\int_{\mathbb{R}} \hat{f} g \, dx = \int_{\mathbb{R}} \hat{g} f \, dx$.

b) Show that the space $L^2(\mathbb{R})$ is not closed w.r.t. convolution. **(4×12=48)**
