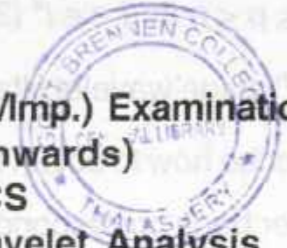




Reg. No. : .....

Name : .....

**Fourth Semester M.Sc. Degree (Reg./Supple./Imp.) Examination, March 2017**  
**(2014 Admission Onwards)**  
**MATHEMATICS**  
**MAT4E05 : Fourier and Wavelet Analysis**



Time : 3 Hours

Max. Marks : 60

- Instructions :**
- 1) Notations are as in **prescribed** text book.
  - 2) Answer **any four** questions from Part – **A**. **Each** question carries **3** marks.
  - 3) Answer **any four** questions from Part – **B** without omitting **any** Unit. **Each** question carries **12** marks.

**PART – A**

1. Define conjugate reflection of  $w \in l^2(\mathbb{Z}_N)$  show that  $(\tilde{w}) \wedge (n) = \overline{w \wedge (n)}$ .
2. Explain how one can generate a  $p^{\text{th}}$  stage wavelet basis. (Make suitable assumptions)
3. Prove that  $l^1(\mathbb{Z})$  is closed w.r.t. convolution.
4. Does there exist a frequency localized orthonormal basis for  $l^2(\mathbb{Z})$  consisting of translates of a vector in  $l^1(\mathbb{Z})$  ?
5. For  $f \in L^1(\mathbb{R})$ , prove that  $(R_y f) \wedge (\xi) = e^{-iy\xi} f \wedge (\xi)$  a.e.
6. Prove or disprove. There is an identity for convolution in  $L^2(\mathbb{R})$ . **(4×3=12)**

**PART – B**

**Unit – 1**

7. a) Show that  $u = \left( \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, 0, 0, 0, 0 \right)$  is a father wavelet.
- b) Find out a mother wavelet  $v$  so that  $u$  and  $v$  generate a first stage wavelet basis.
- c) Write down the signal  $z = (4, 8, -8, 4, 8, 16, 4, -4)$  in terms of the above first stage wavelet basis.



8. Let  $N = 2^n$ ,  $1 \leq p \leq n$  and  $z \in l^2(\mathbb{Z}_N)$ .
- Define a  $p^{\text{th}}$  stage wavelet filter sequence.
  - Briefly describe how to obtain the output of the analysis phase of the  $p^{\text{th}}$  stage wavelet filter bank corresponding to the input  $z \in l^2(\mathbb{Z}_N)$ .
  - Show that the analysis phase of the corresponding  $p^{\text{th}}$  stage wavelet filter can be computed in no more than  $4N + N \log_2 N$  complex multiplications.
9. a) Define up sampling and down sampling operators. If  $N$  is even, show that
- $$(Dz) \wedge (n) = \frac{1}{2} \left[ \hat{z}(n) + \hat{z}\left(n + \frac{N}{2}\right) \right], \text{ for } z \in l^2(\mathbb{Z}_N).$$
- b) For  $z \in l^2(\mathbb{Z}_N)$ , prove that  $(Uz) \wedge (n) = \hat{z}(n)$  for all  $n$ .

### Unit - 2

10. a) Define the spaces  $L^1[-\pi, \pi)$  and  $L^2[-\pi, \pi)$ . Give a norm on each and verify the conditions.
- Show that  $L^2[-\pi, \pi)$  is strictly contained in  $L^1[-\pi, \pi)$ .
  - Prove that point wise convergence doesn't imply norm convergence in  $L^2[-\pi, \pi)$ .
11. a) Show that a bounded translation invariant linear transformation  $T: L^2[-\pi, \pi) \rightarrow L^2[-\pi, \pi)$  is diagonalised by the complex trigonometric system  $\{e^{in\theta}\}_{n \in \mathbb{Z}}$ .
- b) Show that the result in part(a) is not true with real trigonometric system  $\{1\} \cup \{\sqrt{2} \cos n\theta\}_{n \in \mathbb{Z}} \cup \{\sqrt{2} \sin n\theta\}_{n \in \mathbb{Z}}$  by considering the function  $T_j: L^2[-\pi, \pi) \rightarrow L^2[-\pi, \pi)$  defined by  $T_j(f) = \langle f, e^{ij\theta} \rangle e^{ij\theta}$ .



12. a) Suppose  $u \in l^1(\mathbb{Z})$  is such that  $\{R_{2^k}u\}_{k \in \mathbb{Z}}$  is an orthonormal set in  $l^2(\mathbb{Z})$ . Show that this can be extended to a complete orthonormal system (i.e., first stage wavelet system) for  $l^2(\mathbb{Z})$ .
- b) Let  $p \in \mathbb{N}$ . For  $l = 1, 2, \dots, p$  assume that  $u_l, v_l \in l^1(\mathbb{Z})$  are such that the corresponding system matrix  $A_l(\theta)$  is unitary for all  $\theta \in [0, \pi)$ . Suitably defining functions, explain the construction of a  $p^{\text{th}}$  stage wavelet system for  $l^2(\mathbb{Z})$ .

### Unit - 3

13. a) Define convolution of two functions  $f, g: \mathbb{R} \rightarrow \mathbb{C}$  by making suitable assumptions. Establish that it is commutative.
- b) For  $f, g \in L^1(\mathbb{R})$ , show that  $f * g \in L^1(\mathbb{R})$  and that  $(f * g) \wedge = \hat{f} \hat{g}$ .
14. a) State and prove Fourier inversion theorem on  $L^1(\mathbb{R})$ .
- b) Show that if two  $L^1(\mathbb{R})$  functions have same Fourier transforms a.e., the functions are equal a.e.
15. a) For  $f, g \in L^2(\mathbb{R})$ , prove the relation  $\int_{\mathbb{R}} \hat{f} g \, dx = \int_{\mathbb{R}} \hat{g} f \, dx$ .
- b) Show that the space  $L^2(\mathbb{R})$  is not closed w.r.t. convolution. **(4x12=48)**