K17P 0407

Reg. No. : ....

Name : ....

Fourth Semester M.Sc. Degree (Reg./Suppl./Imp.) Examination, March 2017

(2014 Admission Onwards)

MATHEMATICS

MAT 4E 03 : Calculus of Variation

Time : 3 Hours Max. Marks : 60

PART – A

Answer **four** questions from this Part. **Each** question carries **3** marks.

1. Find the extremals of the functional $\int_a^b (y^2 + y'^2 - 2y\sin x)\,dx$.

2. Find the extremals of the functional $\tau[y, z] = \int_0^{\pi/2} (y'^2 + z'^2 + 2yz)\,dx$ subject to conditions $y(0) = 0$, $y(\pi/2) = 1$ $z(0) = 0$, $z(\pi/2) = -1$.

3. Show that for functionals $\tau[y] = \int_{x_0}^{x_1} f(x, y)\sqrt{1 + y'^2}\,dx$ transversality reduces to orthogonality.

4. Find the Hamilton-Jacobi equation corresponding to the functional $\tau[y] = \int_{x_0}^{x_1} f(y)\sqrt{1 + y'^2}\,dx$.

5. Prove that a quadratic functional is differentiable and find its second variation.

6. State three necessary conditions for a functional $\tau[y] = \int_a^b F(x, y, y')\,dx$, $y(a) = A$, $y(b) = B$ to have a weak maximum for the curve $y = y(x)$. **(4×3=12)**

P.T.O.

PART – B

Answer **any four** questions from this Part without omitting **any** Unit. **Each** question carries **12** marks.

Unit – I

7. a) Define variation of a functional and prove that the variation of a differentiable functional is unique.

b) Show that the functional $\tau[y] = \int_a^b y\,dx$ on C [a, b] is differentiable.

c) Prove that a necessary condition for the differentiable functional $\tau[y]$ to have an extremum for $y = \hat{y}$ is that its variation $\delta\tau(b) = 0$ for $y = \hat{y}$ and all admissible h.

8. a) Let $\tau[y] = \int_a^b F(x, y, y')\,dx$ be defined on the set of all continuously differentiable functions y(x) on [a, b] with y (a) = A, y(b) = B. Prove that if $\tau[y]$ has an extremum for a given y(x), then $F_y - \frac{d}{dx}F_{y'} = 0$.

b) Starting from the origin a heavy bead slides along a curve in the vertical plane. Find the curve such that the bead reaches the vertical line x= b in the shortest time.

9. a) Given the functional $J[y] = \int_a^b F(x, y, y')\,dx$, let its admissible curves satisfy the conditions y(a) = A, y(b) = B, $K[y] = \int_a^b G(x, y, y')\,dx = l$, where K[y] is another functional and let $\tau[y]$ have on extremum for y = y (x). If y = y (x) is not an extremal of K[y], prove that there exists a constant λ such that y = y(x) is an extremal of the functional $\int_a^b (F + \lambda G)\,dx$.

b) Among all curves lying on the sphere $x^2 + y^2 + z^2 = a^2$ and passing through two given points (x_0, y_0, z_0) and (x_1, y_1, z_1), find the one which has the least length.

Unit – II

10. a) Derive the basic formula for the general variation of the functional $\tau[y] = \int_{x_0}^{x_1} F(x, y, y')\,dx$.

b) Find the curves for which the functional $\tau[y] = \int_0^{x_1} \frac{\sqrt{1+y'^2}}{y}\,dx$, y(0) = 0 can have extremum if the point (x_1, y_1) can vary along the circle $(x - 9)^2 + y^2 = 9$.

11. a) State and prove Norther's theorem in the variance of the functional $\tau[y] = \int_{x_0}^{x_1} F(x, y, y')\,dx$ under a family of transformation.

b) Use canonical Euler equations to find the extremals of the functional $\int \sqrt{x^2 + y^2}\sqrt{1+y'^2}\,dx$.

12. a) State and prove the principle of least action.

b) Deduce the law of conservation of angular momentum.

Unit – III

13. a) Show that the second variation for the functional $\tau[y] = \int_a^b F(x, y, y')\,dx$ defined for curves y = y(x) with fixed end points y(a) = A, y(b) = B can be expressed as $\delta^2\tau[h] = \int_a^b (Ph'^2 + Qh^2)\,dx$.

b) Prove that a necessary condition for the functional $\tau[y] = \int_a^b F(x, y, y')\,dx$, y(a) = A, y(b) = B to have a minimum for the curve y = y(x) is that $F_{y'y'} \geq 0$.

14. If the quadratic functional $\int_a^b (P h'^2 + Q h^2)\,dx$ where P(x) > 0 ($a \leq x \leq b$) is positive definite for all h(x) with h(a) = 0 = h(b), then prove that [a, b] contains no points conjugate to a.

15. a) If P is a positive definite symmetric matrix and if [a, b] contains no points conjugate to a, then prove that $\int_a^b \left[(P h', h') + (Q h, h)\right] dx$ is positive definite for all h(x) with h(a) = 0 = h(b).

b) Show that the extremals of any functional of the form $\int_a^b F(x, y')\,dx$ have no conjugate points.

(4×12=48)

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