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and (130 = 130 = 130), for all several tendence to along of all Prove that local parametrization of a plane curve is unique upon

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I at C be a connected oriented plane curve and let $\mu = 1 \to C$ be a unit spined

otobel parametrization of G. Prove that it as either one to one or periodic.

b) Define a differential 1-form. When is said to on exact 7 Give an example (with justilization), of a 1-form on ix 2 - (c) when in not exact.

a). For each compact analyzed in surface S in 18th, prove that topic exists a

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by Let S be the hypertropoid of the C. where a, b and mill a D. Findillet

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(a sound to the parametrized forms in \mathbb{R}^2 of $(a,b)=((a+b)\cos b)$.

cos A (a + 0 cost) sin (, sin 4), find the Caussian otherwise of (b.

or State will proof) inverse function theorem for n - surfaces . (4x1)

Reg. No. :	

Fourth Semester M.Sc. Degree (Regular/Supplementary/Improvement)

Examination, March 2016

(2014 Admission)
MATHEMATICS

MAT 4C16: Differential Geometry

Time: 3 Hours Max. Marks: 60

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Answer any four questions. Each question carries 3 marks:

- 1. Show that the graph of any function $f: \mathbb{R}^n \to \mathbb{R}$ is a level set for some function $f: \mathbb{R}^{n+1} \to \mathbb{R}$.
- 2. Let $f: U \to IR$ be a smooth function and let $\alpha: I \to U$ be an integral curve of ∇f . Show that $\frac{d}{dt} (f \circ \alpha) (t) = \|\nabla f(\alpha(t))\|^2$ for all t in I.
- 3. Find the velocity, the acceleration and the speed of the parametrized curve $\alpha(t) = (\cos t, \sin t, t)$.
- 5. Find the length of the parametrized curve $\alpha: [-1, 1] \to \mathbb{R}^3$ defined by $\alpha(t) = (\cos 3t, \sin 3t, 4t)$.
- 6. Obtain the torus as a parametrized 2-surface in IR4.

(4x3=12)



PART-B

Answer any four questions without omitting any Unit. Each question carries 12 marks :

UNIT-I

- a) Let X be a vector field on IR²: X(p) = (p, X(p)) where X(x₁, x₂) = (-x₂, x₁). Find the integral curve of X through p = (a, b).
 - b) Let U be an open set in IRⁿ⁺¹ and let f: U→IR be smooth. Let p ∈ U be a regular point of f and let c = f(p). Prove that the set of all vectors tangent to f⁻¹(c) at p is [∇f(p)][⊥].
 - c) Sketch the level set $f^{-1}(0)$ and typical values $\nabla f(p)$ of the vector field ∇f for $p \in f^{-1}(0)$, where $f(x_1, x_2) = x_1^2 + x_2^2 1$.
- 8. a) Define an n-surface in IRⁿ⁺¹. Give an example with justification.
 - b) State and prove Lagrange multiplier theorem.
 - c) Express the cylinder $x_1^2 + x_2^2 = 1$ in ${\rm I\!R}^3$ as level set of two different functions.
- a) Establish the existence and uniqueness of integral curves of a smooth tangent vector field X on an n-surface S in IRⁿ⁺¹ through p in S.
 - b) Prove that every connected n-surface in IR n+1 has exactly two orientations.
 - c) Find the two orientations of the unit 1-sphere $x_1^2 + x_2^2 = 1$ in \mathbb{R}^2 .

2. Compute V X where w a IR and X are given by X (x, x) = (x, x)

- 10. a) Let S be a compact connected oriented n surface in ℝⁿ⁺¹ with S = f⁻¹(c) where f: ℝⁿ⁺¹ → ℝ is a smooth function such that ∇ f(p) ≠ 0 for all p ∈ S. Prove that the Gauss map N: S → Sⁿ is onto.
 - b) Show that the spherical image of an n-surface with orientation N is the reflection through the origin of the spherical image of the same n-surface with orientation N.



 $(4 \times 12 = 48)$

- 11. a) Let S be an n-surface in \mathbb{R}^{n+1} , let $\alpha:I\to S$ be a parametrized curve in S, let $t_0\in I$ and let $\forall e\in S_{\alpha(t_0)}$. Prove that there exists a unique vector field \bigvee tangent to S along α which is parallel and has $\bigvee_{(t_0)} = \forall r$.
 - b) Let S be an n-surface in \mathbb{R}^{n+1} , let $\alpha: I \to S$ be a parametrized curve, let \mathbb{X} and \mathbb{Y} be vector fields tangent to S along α . Show that i) $(\mathbb{X} + \mathbb{Y})' = \mathbb{X}' + \mathbb{Y}'$ and $(f \mathbb{X})' = f' \mathbb{X} + f \mathbb{X}'$, for all smooth functions f along α .
- 12. a) Prove that the Weingarten map is self adjoint.
 - b) Prove that local parametrization of a plane curve is unique upto reparametrization.
 - c) Find the curvature of the curve $(x_1 a)^2 + (x_2 b)^2 = r^2$, r > 0 oriented by the inward normal vector field.

UNIT - III

- 13. a) Let C be a connected oriented plane curve and let β: I → C be a unit speed global parametrization of C. Prove that β is either one to one or periodic.
 - b) Define a differential 1-form. When is said to be exact? Give an example (with justification) of a 1-form on IR² – {0} which is not exact.
- 14. a) For each compact oriented n-surface S in IRⁿ⁺¹, prove that there exists a point p such that the second fundamental form at p is definite.
 - b) Let S be the hyperboloid $\frac{x_1^2}{a^2} + \frac{x_2^2}{b^2} \frac{x_3^2}{c^2} = 1$, where a, b and c all $\neq 0$. Find the Gaussian curvature K: S \rightarrow IR.
- 15. a) Define differential of a smooth map. Show that it does not depend on the choice of the parametrized curve α .
 - b) Let Φ be the parametrized torus in $\mathbb{R}^3 \Phi$ $(\theta, \phi) = ((a + b \cos \phi), \cos \theta, (a + b \cos \phi) \sin \theta, \sin \phi)$. Find the Gaussian curvature of Φ .
 - c) State (no proof) inverse function theorem for n surfaces.