K16P 0201

Reg. No. : ....

Name : ....

Fourth Semester M.Sc. Degree (Regular/Supplementary/Improvement) Examination, March 2016

MATHEMATICS (2014 Admn.)

MAT 4E05 : Fourier and Wavelet Analysis

Time : 3 Hours Max. Marks : 60

Instructions : *1) Notations are as in prescribed text book.*

*2) Answer **any four** questions from Part **A**. **Each** question carries **3** marks.*

*3) Answer **any four** questions from Part **B** without omitting **any** unit. **Each** question carries **12 marks**.*

PART – A

1. Define a first stage wavelet basis for $l^2(\mathbb{Z}_N)$. Illustrate.

2. Briefly describe how folding lemma can be used in the construction of a p^{th} stage wavelet filter sequence starting with a first stage wavelet basis for $l^2(\mathbb{Z}_N)$.

3. Prove or disprove : A complete orthonormal set of the form $\{R_{2k}w\}_{k\in z}$ exists in $l^2(\mathbb{Z})$ for a suitable $w \in l^1(\mathbb{Z})$.

4. Define a homogeneous wavelet system and p^{th} stage wavelet system for $l^2(\mathbb{Z})$.

5. Define Lebesgue point of a function in $L^1(\mathbb{R})$. Illustrate.

6. Suppose that a function $\mu : \mathbb{R} \to \mathbb{C}$ is multiplicative, not identically zero and differentiable at 0. Prove that $\mu(x) = e^{cx}$ for some $c \in \mathbb{C}$.

PART – B

Unit – 1

7. Express the signal $z = (1,1,0,-1,1,1,0,-1)$ w.r.t. the first stage real Shannon wavelet basis.

8. Suppose N is divisible by 2^l, that $x, y, w \in l^2\left(\mathbb{Z}_{\frac{N}{2^l}}\right)$ and $z \in l^2(\mathbb{Z}_N)$.

Show that $D^l(z) * w = D^l(z * U^l(w))$ and $U^l(x * y) = U^l(x) * U^l(y)$.

P.T.O.

9. a) Briefly describe how to obtain the output of the analysis phase of the p^{th} stage recursive wavelet filter bank.

   b) Suppose $N = 2^n$, $1 \leq p \leq n$ and $u_1, v_1, u_2, v_2 ..., u_p, v_p$ form a p^{th} stage wavelet filter sequence. For an input $z \in l^2 (\mathbb{Z}_N)$, the output $\{x_1, x_2, ..., x_p, y_p\}$ of the analysis phase of the corresponding p^{th} stage wavelet filter can be computed in no more than $4N + N\log_2 N$ complex multiplications.

Unit – 2

10. a) Define $l^2 (\mathbb{Z})$. Prove its completeness.

    b) Prove that norm convergence in $l^2(\mathbb{Z})$ implies pointwise convergence but the converse is not true.

11. Let $v, w \in l^1 (\mathbb{Z})$ and $z \in l^2 (\mathbb{Z})$. Prove the following :

    a) $\widehat{z * w} (\theta) = \hat{z} (\theta) \hat{w} (\theta)$ a.e.

    b) $v * (w * z) = (v * w) * z$.

12. a) Let $u, v \in l^2(\mathbb{Z})$. Define system matrix of u and v. Show that u and v generate a first stage wavelet system for $l^2(\mathbb{Z})$ if and only if the system matrix $A(\theta)$ of u and v is unitary for all $\theta \in [0, \pi)$.

    b) Starting with a $u \in l^1 (\mathbb{Z})$, where $\{R_{2k}u\}_{k \in \mathbb{Z}}$ is orthonormal in $l^2 (\mathbb{Z})$, prove that we can define $v \in l^1 (\mathbb{Z})$ in such a manner that u and v generate a first stage wavelet system for $l^2(\mathbb{Z})$.

Unit – 3

13. Define approximate identity. Suppose that $f \in L^1 (\mathbb{R})$ and that $\{g_t\}_{t>0}$ is an approximate identity. Show that for every Lebesgue point x of f $\lim_{t \to 0^+} g_t * f(x) = f(x)$.

14. a) Show that there is no containing relationship among $L^1 (\mathbb{R})$ and $L^2 (\mathbb{R})$.

    b) Show that $L^1 (\mathbb{R}) \cap L^2 (\mathbb{R})$ is dense in $L^2 (\mathbb{R})$.

15. a) State and prove Fourier inversion theorem on $L^1 (\mathbb{R})$. Interpret the result.

    b) Show that Fourier transform of a function in $L^1 (\mathbb{R})$ is bounded and continuous.
