M 26988

Reg. No. : ....

Name : ....

IV Semester M.A./M.Sc./M.Com. Degree (Reg./Sup./Imp.) Examination, March 2015
MATHEMATICS
Elective : Wavelets

Time: 3 Hours Max. Marks : 60

Instructions : 1) ***Notations*** *are as in prescribed text book.*
2) *Answer* ***any four*** *questions from Part* ***A. Each*** *question carries* ***3*** *marks.*
3) *Answer* ***any four*** *questions from Part* ***B without*** *omitting* ***any*** *Unit.* ***Each*** *question carries* ***12*** *marks.*

PART – A

1. Does there exist a frequency localized orthonormal basis for $l^2(\mathbb{Z}_N)$ formed by the circular translates of a single vector $w \in l^2(\mathbb{Z}_N)$.

2. For z, $w \in l^2(\mathbb{Z}_N)$, show that $(z * w)^\sim = \tilde{z} * \tilde{w}$.

3. Does point wise convergence imply norm convergence in $L^2([-\pi \cdot \pi))$? Justify.

4. For z, $w \in l^1(\mathbb{Z})$ show that $z * w \in l^1(\mathbb{Z})$.

5. If $f \in L^2(\mathbb{R})$ and $g \in L^1(\mathbb{R})$ then show that $f * g \in L^2(\mathbb{R})$.

6. Show that if two functions in $L^1(\mathbb{R})$ have same Fourier transform then the functions are equal a.e.

PART – B
Unit – I

7. a) Write down the first stage Shannon basis for $l^2(\mathbb{Z}_8)$. Show that they form a first stage wavelet basis for $l^2(\mathbb{Z}_8)$. 4

b) Express the signal $z = (1, 0, -1, 0, 1, 0, -1, 0) \in l^2(\mathbb{Z}_8)$. w.r.t. the second stage Shannon wavelet basis. 8

P.T.O.

8. a) Define upsampling and downsampling operators. If N is even, show that

$$\left(\hat{D}z\right)(n) = \frac{1}{2}\left(\hat{z}(n) + \hat{z}\left(n + \frac{N}{2}\right)\right)$$ 7

b) For $z \in l^2(\mathbb{Z}_N)$, prove that $(\mathrm{U}(\hat{z}))(n) = \hat{z}(n)$ for all n. 5

9. a) Explain the construction of Daubechies D_6 wavelets on $\mathbb{Z}_N$. 7

b) Assume N is even. Show that Haar wavelet on $\mathbb{Z}_N$ can be obtained by starting with the identity $\cos^2\left(\frac{\pi n}{N}\right) + \sin^2\left(\frac{\pi n}{N}\right) = 1$. 5

Unit – II

10. a) Suppose $f : [-\pi, \pi) \to \mathbb{C}$ is continuous and bounded. Show that $f(\theta) = 0 \, \forall \theta \in [-\pi, \pi)$ in case $\int_{-\pi}^{\pi} f(\theta)\, e^{-in\theta} d\theta = 0 \, \forall n \in \mathbb{Z}$. 6

b) Suppose that $f \in L^1([-\pi, \pi))$ and that $\int_{-\pi}^{\pi} f(\theta)\, e^{-in\theta} d\theta = 0 \, \forall n \in \mathbb{Z}$. Prove that $f(\theta) = 0$ a.e. 5

c) Deduce that the trigonometric system is complete in $L^2([-\pi, \pi))$. 1

11. Let $w \in l^1(\mathbb{Z})$ and $z \in l^2(\mathbb{Z})$ Prove the following :

a) $(z * \hat{w})(\theta) = \hat{z}(\theta)\, \hat{w}(\theta)$ a.e. 7

b) Show that we may not have $z * w \in l^2(\mathbb{Z})$ even if $z, w \in l^2(\mathbb{Z})$. 5

12. a) Let $u, v \in l^1(\mathbb{Z})$. Show that u and v generate a first stage wavelet system for $l^2(\mathbb{Z})$ if and only if the system matrix $A(\theta)$ of u and v is unitary for all $\theta \in [0, \pi)$. 6

b) If $w \in l^1(\mathbb{Z})$, show that $\{R_k w\}\ k \in \mathbb{Z}$ is a complete orthonormal set in $l^2(\mathbb{Z})$ if and only if $|\hat{w}(\theta)|^2 = 1 \, \forall \theta \in [-\pi, \pi)$ 6

Unit – III

13. a) Define approximate identity. If $\{g_t\}\ t > 0$ is an approximate identity, show that $\lim_{t \to 0+} g_t * f(x) = f(x)$ at every Lebesgue point x of $f \in L^1(\mathbb{R})$. 12

14. a) Show that neither $L^1(\mathbb{R})$ is contained in $L^2(\mathbb{R})$ nor the other way. 8

b) Show that the norm in $L^1(\mathbb{R})$ is not induced by any inner product. 4

15. a) State and prove Fourier inversion theorem for functions in $L^1(\mathbb{R})$. 6

b) For $f \in L^1(\mathbb{R})$, show that $\hat{f}$ is a bounded continuous function on $\mathbb{R}$. 6
