K20P 1188

Reg. No. : ....

Name : ....

III Semester M.Sc. Degree (CBSS – Reg./Suppl./Imp.) Examination, October 2020 (2017 Admission Onwards) MATHEMATICS MAT3C13 : Complex Function Theory

Time : 3 Hours　　　　　　　　　　　　　　　　　　　　　　　　Max. Marks : 80

PART – A

Answer **four** questions from this Part. **Each** question carries 4 marks.

1. Prove that an elliptic function without poles is a constant.
2. Exhibit the Legendre's relation $n_1\omega_2 - n_2\omega_1 = 2\pi i$, where ω_1, ω_2 are periods and n_1, n_2 are constants.
3. Prove that elliptic function without poles is a constant.
4. Whether an analytic function on a region be expressed as limit of a sequence of polynomials. Justify your answer.
5. If u is harmonic, then show that $f = u_x - iu_y$ is analytic.
6. Define Poisson kernel. **(4×4=16)**

PART – B

Answer **any four** questions from this Part without omitting **any** Unit. **Each** question carries **16** marks.

Unit – I

7. a) Prove that a discrete module consists either of zero alone, of the integral multiples $n\omega$ of a single complex number $\omega \neq 0$, or of all linear combinations $n_1\omega_1 + n_2\omega_2$ with integral coefficients of two numbers ω_1, ω_2 with non real ratio ${}^{\omega_2}/_{\omega_1}$.

   b) Prove that the zeroes $a_1, a_2, \ldots, a_n$ and poles $b_1, b_2, \ldots, b_n$ of an elliptic function satisfy $a_1 + \ldots + a_n \equiv b_1 + \ldots + b_n \pmod{M}$.

P.T.O.

8. a) Prove that $\wp(z+u) = -\wp(z) - \wp(u) + \frac{1}{4}\left(\frac{\wp'(z)-\wp'(u)}{\wp(z)-\wp(u)}\right)^2$.

   b) Describe the Modular function $\lambda(\tau)$.

9. a) For Re $z > 1$, prove that $\zeta(z)\Gamma(z) = \int_0^{\infty} (e^t - 1)^{-1} t^{z-1}\, dt$.

   b) State and prove Euler's theorem.

Unit – II

10. State and prove Runge's theorem.

11. Let G be an open connected subset of $\mathbb{C}$ and if G is simply connected then prove that $\mathbb{C}_\infty - G$ is connected.

12. State and prove Schwarz reflection principle.

Unit – III

13. a) Let $u : G \to \mathbb{R}$ be a harmonic function and let $\bar{B}(a; r)$ be a closed disk contained in G. If γ is the circle $|z-a| = r$ then prove that $u(a) = \frac{1}{2\pi}\int_0^{2\pi} u(a + re^{i\theta})\, d\theta$.

    b) State and prove minimum principle.

14. a) If $u : G \to \mathbb{R}$ is a continuous function which has the mean value property then prove that u is harmonic.

    b) State and prove Harnack's theorem.

15. a) Let G be a region and $f : \partial_\infty G \to \mathbb{R}$ a continuous function; then prove that $u(z) = \sup\{\varphi(z) : \varphi \in \mathcal{P}(f, G)\}$ defines a harmonic function u on G.

    b) Derive Jensen's formula. **(4×16=64)**
