K20P 1189

Reg. No. : ....

Name : ....

III Semester M.Sc. Degree (CBSS – Reg./Suppl./Imp.)
Examination, October 2020
(2017 Admission Onwards)
MATHEMATICS
MAT 3C 14 : Advanced Real Analysis

Time : 3 Hours Max. Marks : 80

PART – A

Answer **any four** questions from this Part. **Each** question carries **4** marks.

1. Give an example for a convergent series of continuous functions with discontinuous sum.
2. Suppose $\{f_n\}$ and $\{g_n\}$ converge uniformly on a set E. Show that $\{f_n + g_n\}$ converges uniformly on E.
3. State Parseval's theorem.
4. Show that $\lim_{x\to\infty} x^n e^{-x} = 0$, for every natural number n.
5. Suppose $A \in L(\mathbb{R}^n, \mathbb{R}^m)$.
   a) Define the norm $||A||$ of A.
   b) Show that $|Ax| \leq ||A||\,|x|$ for all $x \in \mathbb{R}^n$.
6. State implicit function theorem. **(4×4=16)**

PART – B

Answer **any four** questions from this Part without omitting **any** Unit. **Each** question carries **16** marks.

Unit – I

7. a) Show that the limit of a uniformly convergent sequence of continuous functions is continuous.
   b) State and prove Weierstrass test for uniform convergence of functions.
   c) Let $f(x) = \sum_{n=1}^{\infty} \frac{1}{x^2 + n^2}$, show that f is continuous on all of $\mathbb{R}$.

8. Suppose f is a continuous complex function on [a, b], then show that there exists a sequence of polynomials P_n such that $\lim_{n\to\infty} P_n(x) = f(x)$ uniformly on [a, b].

9. a) Suppose $\mathscr{B}$ is the uniform closure of an algebra $\mathscr{A}$ of bounded functions. Show that $\mathscr{B}$ is a uniformly closed algebra.

   b) Suppose $\mathscr{A}$ is an algebra of functions on a set E, $\mathscr{A}$ separates points on E and $\mathscr{A}$ vanishes at no point of E. Suppose x_1, x_2 are distinct points of E and c_1, c_2 are constants. Show that $\mathscr{A}$ contains a function f such that $f(x_1) = c_1$ and $f(x_2) = c_2$.

Unit – II

10. a) Suppose $\sum_{n=0}^{\infty} c_n$ converges, define $f(x) = \sum_{n=0}^{\infty} c_n x^n$ for $x \in (-1, 1)$. Show that

$$\lim_{x\to 1} f(x) = \sum_{n=0}^{\infty} c_n.$$

   b) State and prove Taylor's theorem.

11. a) Show that the complex field is algebraically complete.

   b) If f is continuous (with period 2π) and if $\varepsilon > 0$, then show that there is a trigonometric polynomial P such that $|P(x) - f(x)| < \varepsilon$ for all real x.

12. a) Define Gamma function. Show that $\log \Gamma$ is convex on $(0, \infty)$.

   b) Suppose f is a positive function on $(0, \infty)$ such that

      i) $f(x + 1) = x\, f(x)$,

      ii) $f(1) = 1$,

      iii) log f is convex.

      Show that $f(x) = \Gamma(x)$.

Unit – III

13. a) Let r be a positive integer. If a vector space X is spanned by a set of r vectors, then show that $\dim X \leq r$.

   b) Prove that a linear operator A on a finite-dimensional vector space X is one-to-one if and only if the range of A is all of X.

14. Suppose f maps an open set $E \subset \mathbb{R}^n$ into $\mathbb{R}^m$, show that $f \in \mathscr{C}'(E)$ if and only if the partial derivatives $D_j f_i$ exist and are continuous on E for $1 \leq i \leq m$, $1 \leq j \leq n$.

15. State and prove inverse function theorem. **(4×16=64)**