K20P 1191

Reg. No. : ....

Name : ....

III Semester M.Sc. Degree (CBSS – Reg./Suppl./Imp.) Examination, October 2020
(2017 Admission Onwards)
MATHEMATICS
MAT3E02 : Probability Theory

Time : 3 Hours Max. Marks : 80

PART – A

Answer **any four** questions. **Each** question carries **four** marks.

I. a) Define a σ – field and show that every σ – field of subsets of Ω contains the empty set ϕ and the whole space Ω.

b) What is meant by a random variable ? Show that the indicator function I_A is a random variable iff A is an event.

c) Show that $E(X^2) \geq (E(X))^2$; where X a random variable for which both sides of the inequality is are meaningful.

d) Show that a sequence of random variables cannot converge in probability to two essentially different random variables.

e) Show that the characteristic function of a random variable is real iff the random variable is symmetric about the origin.

f) State true or false and justify "The characteristic function exists for every distribution function". **(4×4=16)**

PART – B

Answer **any four** questions without omitting **any** Unit. **Each** question carries **sixteen** marks.

Unit – I

II. a) Explain the concept of the minimal σ – field containing a class of subsets of a given set Ω. Determine the smallest σ – field containing subsets A, B of Ω.

b) Distinguish between a field and a σ – field of subsets of a given set Ω. Give an example of a field which is not a σ – field.

P.T.O.

III. a) If X is a random variable, then show that X^+, X^- and $|X|$ are also random variables.

b) Let $\Omega = \{a, b, c, d\}$ and $\mathscr{A} = \{\phi, \Omega, \{a, b\}, \{c, d\}\}$. Check whether the set function X defined by $X(a) = X(b) = -1$, $X(c) = 1$ and $X(d) = 2$ is a random variable on $(\Omega, \mathscr{A})$.

IV. a) State Caratheodory Extension Theorem on probability measure and illustrate it with an example.

b) Let X be a random variable on the probability space $(\Omega, \mathscr{A}, P)$. For any Borel set $B \in \mathscr{B}$, define $P_X(B) = P[\omega \in \Omega : X(\omega) \in B]$. Show that P_X is a probability measure on $(\mathbb{R}, \mathscr{B})$.

Unit – II

V. a) Define the distribution function of a random variable and illustrate it with an example.

b) State and prove the Jordan Decomposition Theorem for distribution function.

VI. a) State and prove the Holder inequality and derive Schwarz inequality as a special case.

b) Obtain the mean and variance of the gamma distribution.

VII. a) If $X_n \to X$ and $Y_n \to Y$ in probability, then show that $aX_n + bY_n \to aX + bY$ and $X_nY_n \to XY$ in probability.

b) State and prove the monotone convergence theorem on expectation.

Unit – III

VIII. a) Define the characteristic function φ_X of a random variable X and show that it is a continuous function and satisfies $|\varphi(u)| \leq \varphi(0)$.

b) Show that the characteristic function of the Laplace pdf is a constant multiple of Cauchy pdf.

IX. State and prove Bochner's theorem on characteristic function.

X. State and prove Helly-Bray theorem. **(4×16=64)**
