Reg. No. : ....

Name : ....

Third Semester M.Sc. Degree (Reg./Suppl./Imp.)
Examination, November 2017
MATHEMATICS
(2014 Admn. Onwards)
MAT 3C13 : Complex Function Theory

Time : 3 Hours

Max. Marks : 60

PART – A

Answer **any four** questions from this Part. **Each** question carries **3** marks.

1. Show that the sum of residues of an elliptic function is zero.
2. Show that the series $\sum_{n=1}^{\infty} n^{-z}$ converges to an analytic function of z in the half plane $\{z : \operatorname{Re} z > 1\}$.
3. Construct a meromorphic function in the plane with a simple pole at every positive integer n.
4. Define : (i) the complete analytic function obtained from a function element (ii) a complete analytic function.
5. If u is harmonic, show that $f = u_x - iu_y$ is analytic.
6. Let f be an entire function of finite order, show that f assumes each complex number with one possible exception.

PART – B

Answer **any four** questions from this Part without omitting any Unit. **Each** question carries **12** marks.

Unit – I

7. a) Prove that a discrete module consists of either zero alone of the integral multiples nw of a single complex number $w \neq 0$, or all linear combinations

   $n_1 w_1 + n_2 w_2$ with integral coefficients of two numbers w_1, w_2 with nonreal ratio $\frac{w_2}{w_1}$.

   b) Prove that the zeros $a_1, a_2, \ldots, a_n$ and poles $b_1, b_2, \ldots, b_n$ of an elliptic function f(z) satisfy the relation $a_1 + a_2 + \ldots + a_n \equiv b_1 + b_2 + \ldots + b_n \pmod{M}$ where M is the period module of f(z).

P.T.O.

8. a) Establish the formula $P(z) = \frac{1}{z^2} + \sum_{w \neq 0} \left(\frac{1}{(z-w)^2} - \frac{1}{w^2} \right)$ for the weierstrass P-function, where the sum ranges over all $w = n_1 w_1 + n_2 w_2$ except zero.

b) Define Weierstrass zeta function $\zeta(z)$. With usual notations prove the Legendre's relation $\eta_1 w_2 - \eta_2 w_1 = 2\pi i$.

9. a) Prove the Riemann's functional equation for the Riemann zeta function.

b) For Re $z > 1$, prove that $\zeta(z) = \prod_{n=1}^{\infty} \left(1 - P_n^{-z}\right)^{-1}$ where $\{P_n\}$ is the sequence of prime numbers.

Unit – II

10. a) Let K be a compact subset of the region G. Prove that there are straight line segments $\gamma_1, \gamma_2, \ldots, \gamma_n$ in G – K such that for every function f in H(G),

$$f(z) = \sum_{K=1}^{n} \frac{1}{2\pi i} \int_{\gamma_K} \frac{f(w)}{w - z}\, dw \text{ for all z in K.}$$

b) State (without proof) Runge's theorem.

11. a) Let G be an open connected subset of $\mathbb{C}$. If $n(\gamma\,;\,a) = 0$ for every closed rectifiable curve γ in G and every point a in $\mathbb{C}$-G, then prove that $\mathbb{C}_\infty$ – G is connected.

b) State and prove Mittag-Leffler's theorem.

12. a) State and prove the monodromy theorem.

b) Let (f, D) be a function element which admits unrestricted continuation in the simply connected region G. Prove that there is an analytic function $F : G \to \mathbb{C}$ such that $F(z) = f(z)$ for all z in D.

Unit – III

13. a) Define a harmonic function. If $u : G \to \mathbb{C}$ is harmonic , prove that u is infinitely differentiable.

b) State and prove the second version of the maximum principle for Harmonic functions.

c) State (no proof) the minimum principle for harmonic functions.

14. a) State and prove Harnack's inequality.

b) State and prove Jensen's formula.

15. a) Define the genus and order of an entire function.

b) If f is an entire function of finite order λ, prove that f has finite genus $\mu \le \lambda$.

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