K16P 1044

Reg. No. : ....

Name : ....

Third Semester M.A./M.Sc./M.Com. Degree (Reg./Suppl./Imp.) Examination, November 2016
MATHEMATICS
MAT 3C12 : Functional Analysis
(2014 Admission Onwards)

Time : 3 Hours Max. Marks : 60

PART – A

Answer **any four** questions from this **Part**. **Each** question carries **3** marks.

1. Show that the norms $\| \ \|_1$, $\| \ \|_2$ and $\| \ \|_\infty$ on K^n are equivalent.

2. When is a sequence said to be almost convergent ? Give an example with justification.

3. Let X, Y be normed spaces and $F : X \to Y$ be linear. Prove that F is continuous if and only if $g_\circ F$ is continuous for every g in Y'.

4. Show that a continuous map on a normed space is a closed map. Is the converse true ? Justify your answer.

5. Let $\{x_1, \ldots, x_n\}$ be an orthogonal set in an inner product space X. If $k_1, \ldots, k_n$ are scalars with absolute value 1, prove that $\|k_1x_1 + \ldots + k_nx_n\| = \|x_1 + \ldots + x_n\|$.

6. Let X be an inner product space, F a subspace of X and $x \in X$. If $y \in F$ is a best approximation from F to x, then prove that $(x - y) \perp F$. **(4x3=12)**

PART – B

Answer **any four** questions from this Part without omitting **any** Unit. **Each** question carries **12** marks.

Unit – I

7. a) State and prove Riesz Lemma. **6**

   b) Let X be a normed space. Prove that every closed and bounded subset of X is compact if and only if X is finite dimensional. **6**

P.T.O.

8. a) State Hahn-Banach extension theorem (no proof) and give an example of nonunique Hahn-Banach extension. 4

b) Let X be a normed space. For every subspace Y of X and every $g \in Y'$. Prove that there is a unique Hahn-Banach extension of g to X if and only if X' is strictly convex. 8

9. a) Prove that a normed space X is a Banach space if and only if every absolutely summable series of elements of X is summable in X. 6

b) Let X and Y be normed spaces and $X \neq \{0\}$. Prove that BL(X, Y) is a Banach space in the operator norm if and only if Y is a Banach space. 6

Unit – II

10. a) Let X be a normed space and E be a subset of X. Prove that E is bounded in X if and only if f(E) is bounded in K for every f in X'. 5

b) Let $X = \{x \in C([-\pi, \pi]) : x(\pi) = x(-\pi)\}$ with sub norm. Prove that the Fourier series of every x in a dense subset of X diverges at 0. 7

11. a) Let X and Y be Banach spaces and $F : X \to Y$ be a closed linear map. Prove that F is continuous. 8

b) Show that the result in part (a) may not hold if X and Y are not Banach spaces. 4

12. a) State the two-norm theorem (no proof). 2

b) Prove that the coefficient functionals corresponding to a Schauder basis for a Banach space are continuous. 10

Unit – III

13. a) Prove that an inner product <, > on a linear space X induces a norm on X. 3

b) State a characterization of an inner product space among all normed spaces. 2

c) State and prove Bessel's inequality. 7

14. a) Let $\{u_\alpha\}$ be an orthonormal set in a Hilbert space H. Prove that $\{u_\alpha\}$ is an orthonormal basis for H if and only if span $\{u_\alpha\}$ is dense in H. 6

b) Let E be a nonempty closed convex subset of a Hilbert space H. Prove that for each x in H, there exists a unique best approximation from E to x. 6

15. a) State Riesz representation theorem (no proof). What does it state in the case of the Hilbert space K^n ? Justify your answer. 3

b) Show that Riesz representation theorem does not hold for an incomplete inner product space. 3

c) Let H be a Hilbert space, G a subspace of H and g be a continuous linear functional on G. Prove that there is a unique continuous linear functional f on H such that $f|_G = g$ and $\|f\| = \|g\|$. 6

(4×12=48)
