Reg. No. : ...................................

Name : ........................................

# Third Semester M.A./M.Sc./M.Com. Degree (Reg./Suppl./Imp.) Examination, November 2016
## MATHEMATICS
### (2014 Admission Onwards)
### MAT 3C13 : Complex Function Theory

Time : 3 Hours Max. Marks : 60

PART – A

Answer **any four** questions from this Part. **Each** question carries **3** marks.

1. Define the Weirstrass functions $\zeta(z)$ and $\sigma(z)$. Also show that

$$\sigma(z + w_1) = - \sigma(z)\ e^{\eta_1(z + w_1/2)}.$$

2. Prove that $\frac{\zeta'(z)}{\zeta(z)} = -\sum_{n=1}^{\infty} \frac{\Lambda(n)}{n^z}$ for Rez > 1, where $\zeta(z)$ is the Reimann zeta function and $\Lambda(n)$ is the Mangoldt's function.

3. Show that $\mathbb{C}$ and $D = \{z : |z| < 1\}$ are homeomorphic.

4. Define i) function element ii) analytic continuation along a path.

5. Show that any two harmonic conjugates of a given harmonic function differ by a real constant.

6. If f is an entire function of finite order, then prove that f assumes each complex number with one possible exception.

P.T.O.

PART – B

Answer **any four** questions from this Part without omitting any Unit. **Each** question carries **12** marks.

**Unit – I**

7. a) Prove that any two bases of the same period module are connected by a unimodular transformation.

   b) Prove that an elliptic function without poles is constant.

   c) If $a_1, \ldots a_n$ are zeros $b_1, \ldots, b_n$ are poles of an elliptic function, prove that $(a_1 + \ldots\ldots + a_n) - (b_1 + \ldots + b_n)$ is a period.

8. a) With usual notations prove that $\wp'(z)^2 = 4\wp(z)^3 - g_2\wp(z) - g_3$. Deduce that $\wp(z)$ is an inverse of an elliptic integral.

   b) Establish the addition theorem for the $\wp$ – function.

9. a) Define the Riemann zeta function for Rez>1, prove that

   $\zeta(z)\Gamma(z) = \int_0^\infty (e^t - 1)^{-1} t^{z-1} dt$.

   b) If Rez > 1, prove that $\zeta(z) = \prod_{n=1}^{\infty}(1 - P_n^{-z})^{-1}$, where $\{P_n\}$ is the sequence of prime numbers.

**Unit – II**

10. Let K be a compact subset of the region G. Prove that there are straight line segments $r_1, r_2, \ldots, r_n$ in G – K such that for every function f in H(G) $f(z) = \sum_{k=1}^{n} \frac{1}{2\pi i} \int_{r_k} \frac{f(w)}{w - z} dw$.

11. a) State and prove Mittag – Leffler's theorem.

    b) Construct a meromorphic function with a simple pole at every integer n.

12. State and prove Schwarz Reflection Principle.



**Unit – III**

13. a) State and prove the mean value theorem for harmonic functions.

    b) State and prove the maximum principle (second version) for harmonic functions.

14. Prove that the Dirichlet problem can be solved for the unit disc.

15. a) State and prove Jensen's formula.

    b) Define order of an entire function. Find the order of $f(z) = \exp(z^2)$.

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