b) If the differential equation $y' = x^2 + y^2 - 2$ satisfies the following data :

x	y
– 0.1	1.0900
0	1.0000
0.1	0.8900
0.2	0.7605

Use Milne's method to obtain the value of y (0.3).

Reg. No. : ....

Name : ....

III Semester B.Sc. Hon's (Maths) Degree (Reg./Supple./Improve.) Examination, November 2015
BHM 303 : DIFFERENTIAL EQUATIONS

Time : 3 Hours Max. Marks : 80

Answer **all** the **ten** questions. **(10×1=10)**

1. Verify that $y = e^x + ax^2 + bx + c$ is a solution of $y''' = e^x$.

2. Solve : $y' - y = 4$.

3. State the existence theorem for the solution of the initial value problem $y' = f(x, y),\ y(x_0) = y_0$.

4. If $y_1 = \cos wx$, $y_2 = \sin wx$ are solutions of $y'' + w^2y = 0$, obtain their Wronskian.

5. Give the formula for the particular solution of the differential equation $y'' + p(x)\ y' + q(x)\ y = r(x)$, where p(x), q(x), r(x) are continuous in some open interval I.

6. Give the condition for the differential equation P (x, y) dx + Q (x, y) dy to be exact.

7. Find an integrating factor of 2 cos hx cos y dx = sin hx sin y dy.

8. Find the differential equation of the orthogonal trajectories of $y' = f(x, y)$.

9. Give the general solution if the characteristic equation of the differential equation $y'' + ay' + by = 0$ has equal roots.

10. Obtain the auxiliary equation of the Euler – Cauchy equation $x^2 y'' + axy' + by = 0$.

P.T.O.

Answer **any 10** short answer questions out of **14**. **(10×3=30)**

11. Solve : $xy^1 = y^2 + y$.

12. Solve : $y' + y = e^{-x} \tan x$.

13. Find the orthogonal trajectories of $y = ce^{-x}$.

14. Show that $\frac{1}{xy}$ is an integrating factor of $(2y + xy)\,dx + 2x\,dy = 0$ and solve it.

15. Show that any linear combination of two solutions of the homogeneous linear differential equation $y'' + p(x)\,y' + q(x)\,y = 0$ on an open interval I, is again a solution.

16. Solve the initial value problem $9y'' + 6y' + y = 0$, $y(0) = 4$, $y'(0) = -13/3$.

17. Find a general solution of the equation $4x^2y'' + 12\,xy' + 3y = 0$.

18. Show that the difference of two solutions of the equation $y'' + p(x)\,y' + q(x)\,y = r(x)$, $r(x) \neq 0$ on some open interval I is also a solution of the equation $y'' + p(x)\,y' + q(x)\,y = 0$, where p (x) and q (x) are continuous variable constants.

19. Determine the type and stability of the critical point of the system $y_1' = -2y_1 + y_2$, $y_2' = -2y_1 - 2y_2$.

20. If $y^1 = x + y^2$, find $y^{(1)}$ using Picard's iteration method, $y(0) = 1$.

21. State the fourth order Runge-Kutta formula for solving the differential equation $y' = f(x, y)$, $y(x_0) = y_0$.

22. Given $\frac{dy}{dx} = y - x$, $y(0) = 2$, find y (0.1) correct to three decimal places by second order Runge-Kutta method.

23. State Adam's Predictor-Corrector formula for solving the initial value problem $y' = f(x, y)$, $y(x_0) = y_0$.

24. Solve the equation $y'' - 9y = 0$ by converting it to two first order equations.

Answer **any six** short essay questions. **(6×5=30)**

25. Solve : $(1 + y + 2x)\,y' = 1 - 2y - 4x$.

26. Solve : $y' + \frac{1}{3}y = \frac{1}{3}(1 - 2x)\,y^4$.

27. If $y^1 = x - y^2$, $y(0) = 1$, find y (0.1) correct to three decimal places by Taylor series.

28. Reduce to first order and solve the differential equation $x^2y'' - xy' + y = 0$, where $y_1 = x$ is one solution.

29. Verify $y_p = xe^x$ is a solution of the equation $y'' - y = 2e^x$ and solve the initial value problem $y'' - y = 2e^x$, $y(0) = -1$, $y'(0) = -1$.

30. Solve : $y'' + 4y = 8x^2$.

31. Solve : $y'' - 4y' + 4y = e^{2x}/x$.

32. Find a general solution of the system of equations $y_1' = -3y_1 + y_2$, $y_2' = y_1 - 3y_2$.

33. If $y' = -y$, $y(0) = 1$, find y (0.01) and y (0.02) by Euler's method (h = 0.01).

Answer **any one** essay question out of **two**. **(1×10=10)**

34. If the auxiliary equation of the Euler-Cauchy equation $x^2y'' + axy' + by = 0$ has double root, show that the general solution of Euler Cauchy equation is given by $y = (c_1 + c_2 \ln x)\,x^{\frac{1}{2}(1-\alpha)}$.

35. a) State Milne's predictor – Corrector formula for the solution of the problem

$$y' = f(x, y),\ y(x_0) = y_0.$$