K15P 0061

Reg. No. : ....

Name : ....

Third Semester M.A./M.Sc./M.Com. Degree (Reg./Sup./Imp) Examination, November 2015
MATHEMATICS (2014 Admn.)
MAT 3E01 : Graph Theory

Time : 3 Hours   Max. Marks : 60

PART – A

Answer **any 4** questions. **Each** question carries **3** marks :

I. 1) Find the covering number and edge covering number of the complete graph K_n.

2) Prove that no graph has chromatic polynomial $k^4 - 3k^3 + 3k^2$.

3) If G is a complete bipartite graph, then prove that $\chi'(G) = \Delta(G)$.

4) Show that if G is a simple planar graph such that G is isomorphic to its dual G*, then $\epsilon = 2\upsilon - 2$.

5) Prove or disprove : A connected graph G has a perfect matching if and only if $|N(s)| \geq |s|$ for all $s \leq v$.

6) If u and v are two non-adjacent vertices of the Peterson graph, find the minimum number of vertices in a u – v separating set. Verify the Menger's theorem.

PART – B

Answer **any 4** questions without omitting **any** Unit. **Each** question carries **12** marks :

Unit – I

II. a) If α and β are respectively the independence number and covering number of G, prove that $\alpha + \beta = \gamma(G)$. 5

P.T.O.

b) Prove that for any two positive integers k and l, $r(k, l) \leq \binom{k+l-2}{k-1}$. **4**

c) Establish that $r(2, l) = l$. **3**

III. a) Prove that if $(s_1, s_2, ..., s_n)$ is a partition of the set of integers $\{1, 2, ..., r_n\}$, then for some i, s_i contains x, y and z such that $x + y = z$. (Here r_n denote the Ramsey number $r(k_1, ..., k_n)$, with $k_i = 3$ for all i). **5**

b) Prove that if G is k-critical, then $\delta \geq k - 1$. **4**

c) Prove that every critical graph is connected. **3**

IV. a) Prove that for any positive integer k, these exists a k-chromatic graph containing no triangle. **8**

b) Let G be a simple graph. Prove that the coefficient of $k^{\upsilon - 1}$ is $\pi_k(G)$ is $-\epsilon$. **4**

Unit – II

V. a) Let G be a connected graph that is not an odd cycle. Prove that G has a 2-edge colouring in which both colours are represented at each vertex of degree atleast 2. **8**

b) What is time tabling problem ? Explain how it can be solved using the theory of edge coloring. **4**

VI. a) State and prove the Euler's formula. **8**

b) Explain how k_5 is embeddable on a torus ? **4**

VII. a) Define bridge of a subgraph. Prove that if two bridges overlap, then either they are skew or else they are equivalent 3- bridges. **8**

b) Show that if G is a simple planar graph with $\upsilon \geq 11$, then G^* cannot be planar. **4**

Unit – III

VIII. a) Prove that a graph G has a perfect matching if and only if $0(G - S) \leq |S|$ for all $S \subset V$. Explain how this result is true in case of complete graphs of odd number of vertices. **10**

b) Give an example of a non-regular bipartile graph having a perfect matching. **2**

IX. What is the personal assignment problem ? Describe a method to solve it. Illustrate with an example. **12**

X. State and prove the Menger's theorem. **12**