K15P 0060

Reg. No. : ....

Name : ....

Third Semester M.A./M.Sc./M.Com. Degree (Reg./Sup./Imp.)
Examination, November 2015
MATHEMATICS (2014 Admn.)
MAT3C14 : Advanced Real Analysis

Time : 3 Hours Max. Marks : 60

***Instructions** : Answer **4** questions from Part – **A**. **Each** question carries 3 marks.*
*Answer **any 4** questions from Part – **B** without omitting any Unit.*
*Each question carries **12** marks.*

PART – A

1. Show by an example that the limit of a sequence of Riemann integrable functions need not be Riemann integrable.

2. Prove that every uniformly convergent sequence of bounded functions is uniformly bounded.

3. Define gamma function Γ and prove that $\Gamma(n+1) = n!$ for $n = 1, 2 ....$

4. If z is a complex number with $|z|=1$, prove that there is a unique t in $(0, 2\pi)$ such that $E(it) = z$.

5. Prove that to every $A \in L(\mathbb{R}^n, \mathbb{R}^1)$ corresponds a unique $y \in \mathbb{R}^n$ such that $Ax = x.y$. Prove also that $||A|| = ||y||$.

6. Suppose f is a differentiable mapping of $\mathbb{R}^1$ into $\mathbb{R}^2$ such that $|f(t)|=1$ for every t. Prove that $f'(t).f(t) = 0$.

PART – B

UNIT – I

7. a) State and prove Cauchy's criterion for uniform convergence.

   b) If $\{f_n\}$ is a sequence of continuous functions on E and if $f_n \to f$ uniformly on E then prove that f is continuous on E.

P.T.O.

8. a) Prove that the space $\mathbb{C}(x)$ of all continuous complex bounded functions on a metric space X with the supremum norm is a complete metric space.

b) Suppose $\{f_n\}$ is an equicontinuous sequence of functions on a compact set K and $\{f_n\}$ converges pointwise on K. Prove that $\{f_n\}$ converges uniformly on K.

9. a) Suppose K is compact and $\{f_n\}$ is a sequence of continuous functions on K which converges pointwise to a continuous function f on K. If $f_n(x) \geq f_{n+1}(x)$ for all $x \in K$ and n = 1, 2,...then prove that $f_n \to f$ uniformly on K. Show by an example that the assumption compactness of K cannot be dropped.

b) Suppose $\mathscr{A}$ is a self adjoint algebra of complex continuous functions on a compact set K. $\mathscr{A}$ separates points on K. Prove that $\mathscr{A}$ is dense in $\mathbb{C}(k)$.

UNIT – II

10. a) For a double sequence $\{a_{ij}\}$, i = 1, 2, 3 = ..., j = 1, 2, 3 ..... suppose that

$$\sum_{j=1}^{\infty} | a_{ij} | = b_i \ (i = 1,2,3..) \text{ and } \sum_{i=1}^{\infty} b_i \text{ converges then prove that } \sum_{i=1}^{\infty}\sum_{j=1}^{\infty} a_{ij} = \sum_{j=1}^{\infty}\sum_{i=1}^{\infty} a_{ij}.$$

b) Suppose $f(x) = \sum_{n=0}^{\infty} c_n x^n$, the series converges in $|x| < R$. If $-R < a < R$ then prove that f can be expanded in a power series about the point x = a which converges in $|x - a| < R - |a|$ and also prove that $f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)(a)}}{n!}(k-a)^n$ $(1x - a| < R - |a|)$.

11. a) Let f be a 2π periodic function defined on $\mathbb{R}$. Suppose f is Riemann integrable on $[-\pi, \pi]$. If for some x, there are constants $\delta > 0$ and $M < \infty$ such that $| f(x+t) - f(x) | \leq M | t |$ for all $t \in (-8, 8)$ then prove that $\lim_{N \to \infty} S_N(f; x) = f(x)$ where $S_N(f; x)$ is N^{th} partial sum of the Fourier series of f.

b) If f is continuous (with period 2π) on $\mathbb{R}$ and if $\epsilon > 0$ prove that there is a trigonometric polynomial p such that $| p(x) - f(x) | < \epsilon$ for all real x.

12. a) State and prove Parseval's theorem.

b) If x > 0 and y > 0 then prove that $\int_0^1 t^{x-1}(1+t)^{y-1} dt = \frac{\Gamma(x)\ (y)}{\Gamma(x+y)}$.

UNIT – III

13. a) Prove that a linear operator A on a finite dimensional vector space X is one-to-one if and only if the range of A is all of X.

b) Prove that the set Ω of all inversible linear operators on $\mathbb{R}^n$ is open in $L(\mathbb{R}^n)$ and the mapping $A \to A^{-1}$ from Ω to Ω is continuous.

14. a) Define differentiability of a function f from an open set $E \subset \mathbb{R}^n$ into $\mathbb{R}^m$ and prove that the derivative of at a point of $\in$ exists then it is unique.

b) State and prove chain rule for differentiation for functions of sereval variables.

15. a) Suppose f maps an open set $E \subset \mathbb{R}^n$ into $\mathbb{R}^m$ and f is differentiable at a point $x \in E$. Then prove that the partial derivatives $(D_j f_i)(x)$ exists and also prove that $f'(x)e_j = \sum_{i=1}^{m} (D_j f_i)(x) u_i \quad (1 \leq j \leq x)$.

b) Suppose f maps a convex open set $E \subset \mathbb{R}^n$ into $\mathbb{R}^m$. If $f'(x) = 0$, for all $x \in E$ then prove that f is a constant.