K15P 0059

Reg. No. : ....

Name : ....

Third Semester M.A./M.Sc./M.Com. Degree (Reg./Sup./Imp.) Examination, November 2015
MATHEMATICS (2014 Admn.)
MAT 3C13 : COMPLEX FUNCTION THEORY

Time : 3 Hours Max. Marks : 60

PART – A

Answer **any four** questions from this Part. **Each** question carries **3** marks.

1. Prove the Legendre's relation $\eta_1 w_2 - \eta_2 w_1 = 2\pi i$.

2. Prove that the series $\sum_{n=1}^{\infty} n^{-z}$ converges to an analytic function in the half plane $\text{Re}z > 1$.

3. Construct a meromorphic function in the plane with a simple pole at each integer n.

4. Define (i) function element (ii) analytic continuation along a path.

5. If u is harmonic in a region G, prove that $f(z) = u_x - i\,u_y$ is analytic.

6. Let f be an entire function of finite order λ, where λ is an integer. Then prove that f has infinitely many zeros.

PART – B

Answer **any four** questions from this Part without omitting **any** Unit. **Each** question carries **12** marks.

UNIT – I

7. a) Prove that a non zero discrete module consists either of the integral multiples of a single complex number $w \neq 0$ or all linear combinations $n_1 w_1 + n_2 w_2$ with integral coefficients of two numbers w_1, w_2 with nonreal ratio w_2/w_1.

   b) Prove that the sum of residues of an elliptic function is zero.

P.T.O.

8. a) With usual notations prove that $\wp(z)=\frac{1}{z^2}+\sum_{w\neq 0}\left(\frac{1}{(z-w)^2}-\frac{1}{w^2}\right)$, where the sum ranges over all $w = n_1w_1 + n_2w_2$ except zero.

   b) Prove that $\wp(z+u)+\wp(z)+\wp(u)=\frac{1}{4}\left(\frac{\wp'(z)-\wp'(u)}{\wp(z)-\wp(u)}\right)^2$.

9. If $\operatorname{Re} z > 1$ and ζ denotes the Riemann zeta function prove that

   i) $\zeta(z)\Gamma_{(z)}=\int_0^\infty\left(e^t-1\right)^{-1}t^{z-1}\,dt$

   ii) $\zeta(z)=\prod_{n=1}^{\infty}\left(\frac{1}{1-P_n^{-z}}\right)$, where $\{P_n\}$ is the sequence of prime numbers.

UNIT – II

10. State and prove Runge's theorem. Include all necessary lemma's with proof.

11. a) Prove that $\mathbb{C}$ and $D = \{z : |z| < 1\}$ are homeomorphic.

    b) State and prove Mittag-Leffler's theorem.

12. a) State and prove the Monodromy theorem.

    b) Let (f, D) be a function element which admits unrestricted continuation in the simply connected region G. Prove that there is an analytic function $F : G \to \mathbb{C}$ such that $F(z) = f(z)$ for all z in D.

UNIT – III

13. a) State and prove the first version of maximum principle for harmonic functions.

    b) Define the Poisson Kernel and derive three of its properties.

14. a) If $u : \bar{B}(a; R) \to \mathbb{R}$ is continuous, harmonic in $B(a; R)$ and $u \geq 0$, prove that for $0 \leq r < R$ and all θ, $\frac{R-r}{R+r}u(a) \leq u\left(a+re^{i\theta}\right) \leq \frac{R+r}{R-r}u(a)$.

    b) Let G be a region. If $\{u_n\}$ is a sequence in Har(G) such that $u_1 \leq u_2 \leq \ldots$, then prove that either $u_n(z) \to \infty$ uniformly on compact subsets of G or $\{u_n\}$ converges in Har(G) to a harmonic function.

15. a) Define rank, genus and order of an entire function.

    b) If f is an entire function of finite order λ then prove that f has finite genus $\mu \leq \lambda$.

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