K15P 0057

Reg. No. : ....

Name : ....

Third Semester M.A./M.Sc./M.Com. Degree (Reg./Supple./Imp.) Examination, November 2015
MATHEMATICS (2014 Admn.)
MAT 3C11 : Number Theory

Time : 3 Hours Max. Marks : 60

PART – A

Answer **any four** questions from this Part. **Each** question carries **3** marks.

1. Prove that d(n) is odd if and only if n is a square.

2. If $P \geq 5$ is a prime and $1+\frac{1}{2}+\frac{1}{3}+...+\frac{1}{P}=\frac{r}{P^5}$, prove that $P^3 \mid (r-s)$.

3. Prove that the Legendre symbol $(n \mid p)$ is a completely multiplicative function of n.

4. Find the solution of the knapsack problem $51 = 3x_1 + 5x_2 + 9x_3 + 18x_4 + 37x_5$.

5. Define the norm and trace of an element α in a number field K. If [K : Q] = n and $\alpha \in Q$, find $N(\alpha)$ and $T_K(\alpha)$.

6. Find integral basis and discriminant of $Q(\sqrt{5})$.

PART – B

Answer **any four** questions from this Part without omitting **any** Unit. **Each** question carries **12** marks.

Unit – I

7. a) If p_n is the n^{th} prime, prove that the infinite series $\sum_{n=1}^{\infty} \frac{1}{P_n}$ diverges.

   b) State the Euclidean algorithm. Use it to find the gcd of 826 and 1890 and express the gcd as a linear combination of 826 and 1890.

P.T.O.

PART – B

Answer **any four** questions from this Part without omitting any Unit. **Each** question carries **12** marks.

UNIT – I

7. a) Let X be a normed space. Prove that every closed and bounded subset of X is compact if and only if X is finite dimensional. 6

   b) Let X and Y be normed spaces and $F : X \to Y$ be a linear map such that the range R(F) is finite dimensional. Prove that F is continuous if and only if the zero space Z(F) of F is closed in X. 6

8. a) State and prove Hahn-Banach extension theorem. 7

   b) Give an example of non unique Hahn-Banach extension. 4

   c) State a characterization (without proof) of normed spaces which admit unique Hahn-Banach extensions. 1

9. a) Prove that a normed space X is a Banach space if and only if every absolutely summable series of elements in X is summable in X. 6

   b) Prove that a Banach space cannot have a denumerable Hamel basis. 3

   c) Define a Schauder basis for a normed space and give an example. 3

UNIT – II

10. a) State and prove uniform boundedness principle. 8

    b) Show that uniform boundedness principle may not hold if the domain space is not Banach. 4

11. a) Let X and Y be Banach spaces and $F : X \to Y$ be a closed linear map. Prove that F is continuous. 8

    b) Let P be a projection on a normed space X such that R(P) and Z(P) are closed in X. Prove that P is a closed map. 4

12. a) Let X and Y be Banach spaces and let $F \in BL(X, Y)$ be bijective. Prove that $F^{-1} \in BL(Y, X)$. 3

    b) If $k_n \in K$, $n = 0, \pm 1, \pm 2, \ldots$ such that $k_n \to 0$ as $n \to \pm\infty$, does there exist some $x \in L^{-1}([-\pi, \pi])$ such that $\hat{x}(n) = k_n$ for each n ? Justify your answer. 9

UNIT – III

13. a) State and prove Schwarz inequality on an inner product space. 6

    b) Prove that an $\langle \, , \, \rangle$ on a linear space X induces a norm on X. 3

    c) Among all the l^p spaces $1 \le p < \infty$ show that only l^2 is an inner product space. 3

14. a) Let H be a nonzero Hilbert space over K. Prove that H has a countable orthonormal basis if and only if H is separable. 6

    b) Let E be a nonempty closed convex subset of a Hilbert space H. For each x in H, prove that there exists a unique best approximation from E to x. 6

15. a) State and prove projection theorem. 6

    b) Let (x_n) be a sequence in a Hilbert space H. If (x_n) is bounded, then prove that it has a weak convergent subsequence. 6