Reg. No.:....

Name :

Third Semester M.A./M.Sc./M.Com. Degree (Reg./Sup./Imp.)
Examination, November 2014
(2007 Admn. Onwards)
MATHEMATICS

Paper - XII: Complex Analysis

Time: 3 Hours

Max. Marks: 60

PART-A

Answer any four questions. Each question carries 3 marks.

- 1. Compute $\int_{|z|=2}^{\infty} \frac{dz}{z^2+1}$ by decomposition of the integrand into partial fractions.
- 2. How many roots of the equation $z^4 6z + 3 = 0$ have their modulus between 1 and 2?
- 3. Define a harmonic function. If u is harmonic in a region Ω show that $f(z) = \frac{\partial u}{\partial x} i \frac{\partial u}{\partial y} \text{ is analytic in } \Omega \,.$
- 4. Show that the series $\zeta(z) = \sum_{n=1}^{\infty} n^{-z}$ converges for Re z > 1 and represent the derivative in series form.
- 5. Develop the function $f(z) = (1+z)^{\mu}$ is Taylor series about the origin, choosing a branch of f(z) which equals 1 at the origin.
- 6. Prove that an elliptic function without poles is constant.

 $(4 \times 3 = 12)$

PART-B

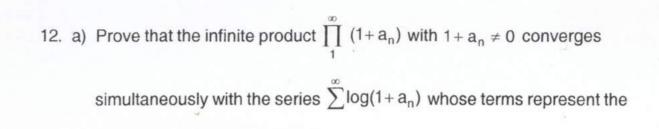
Answer any four questions without omitting any Unit. Each question carries 12 marks.

Third Semester M.A.M.Sc./I_TINU Degree (Reg./Sup./Imp.)

- 7. a) State and prove Cauchy's theorem for a rectangle.
 - b) Prove that a function which is analytic and bounded in the whole plane must reduce to a constant.
- 8. a) Prove that an analytic function comes arbitrarily close to any complex value in every neighborhood of an essential singularity.
 - b) If f(z) is analytic and nonconstant in a region Ω , prove that |f(z)| has no maximum in Ω .
 - c) State and prove lemma of Schwarz.
- 9. a) State and prove the residue theorem.
 - b) Evaluate $\int_{-\infty}^{\infty} \frac{x^2 x + 2}{x^4 + 10x^2 + 9} dx$ by the method of residues.

UNIT-I

- 10. a) Prove that the arithmetic mean of a harmonic function over concentric circles $|z|=r \text{ is a linear function of } \log r, \ \frac{1}{2\pi} \ \int\limits_{|z|=r}^{} u \ d\theta = \alpha \log r + \beta \ .$
 - b) Prove that the function $P_U(z)$ is harmonic for |z| < 1 and $\lim_{z \to e^{i\theta_0}} P_u(z) = U(\theta_0)$.
- 11. a) State and prove Mittag-Leffler theorem.
 - b) Comparing the coefficients in the Laurent developments $\cot \pi z$ and of its expression as a sum of partial fractions find the value of $\sum_{1}^{\infty} \frac{1}{n^2}$.



b) Derive the factorization $\sin \pi z = \pi z \, \prod_1^\infty \Biggl(1 - \frac{z^2}{n^2} \Biggr).$

principal branch of the logarithm.

UNIT - III

- 13. a) Prove that the ζ -function can be extended to a monomorphic function in the whole plane with a simple pole at s = 1 with the residue 1.
 - b) Prove the functional equation:

$$\zeta(s) = 2^{s} \pi^{s-1} \sin \frac{\pi s}{2} \Gamma(1-s) \zeta(1-s)$$
.

- 14. a) Prove that a discrete module consists either zero alone, of the integral multiples nw of a single complex number $w \neq 0$, or all linear combinations $n_1w_1 + n_2w_2$ with integral coefficients of two numbers with non real ratio $\frac{w_2}{w_1}$.
 - b) Prove that any two bases of the same module are connected by a unimodular transformation.
- O15. a) Define the Weirstrass's function $\zeta(z)$ and prove that $\zeta(z+w_1)=\zeta(z)+\eta_1\,;\, \zeta(z+w_2)=\zeta(z)+\eta_2$ where η_1 and η_2 are constants. Also prove that the quantities $\eta_1,\,\eta_2,\,w_1,\,w_2$ are connected by the relation $\eta_1w_2-\eta_2w_1=2\pi i$.
 - b) With usual notations prove that the Weirstrass \wp function satisfies the differential equation $\wp^1(z)^2 = 4\wp(z)^3 = g_2\wp(z) g_3$. (4×12=48)