M 26092

Reg. No. : ....

Name : ....

Third Semester M.A./M.Sc./M.Com. Degree (Reg./Sup./Imp.) Examination, November 2014
(2007 Admn. Onwards)
MATHEMATICS
Paper – XIV : Elective – I : Graph Theory
(2009 Admission)

Time: 3 Hours — Max. Marks : 60

PART – A

(Answer **any four** questions. **Each** question carries **3** marks).

1. Define Ramsey number r (k, *l*). Show that for all k, r (k, 2) = k.
2. Define chromatic number X(G) of a graph G. Give an example of a 3-chromatic graph with justification.
3. Obtain the edge chromatic number of the Petersen graph.
4. What is meant by planar embedding of a graph ? Illustrate with an example.
5. Show that every tournament is either disconnected or can be transformed into a disconnected tournament by the reorientation of just one arc.
6. In a network define (i) flow (ii) resultant flow (iii) maximum flow. **(4×3=12)**

P.T.O.

PART – B

Answer **any four** questions without omitting **any** unit. **Each** question carries 12 marks.

Unit – I

7. a) Define the parameters α' and β' for a graph and if $\delta > 0$, then prove that $\alpha' + \beta' = \upsilon$.

   b) Prove that :

   i) $r(k, l) \le \binom{k+l-2}{k-1}$ ii) $r(k, k) \ge 2^{k/2}$.

8. a) If a simple graph G contains no K_{m+1}, prove that G is degree majorized by some complete m – partite graph H. Further show that, if G has the same degree sequence as H, then $G \cong H$.

   b) If G is simple and contains no K_{m+1}, then prove that $\sum(G) \le \sum(T_{m,\upsilon})$. Further prove that equality holds only if $G \cong T_{m,\upsilon}$.

9. a) If G is 4-chromatic, prove that G contains a subdivision of K_4.

   b) For any positive integer k, prove that there is a k-chromatic graph containing no triangle.

Unit – II

10. a) If G is a simple graph prove that $\chi = \Delta$ or $\chi' = \Delta + 1$.

    b) If G is bipartite and $p \ge \Delta$, prove that there exist p disjoint matchings $M_1, M_2, \ldots, M_p$ of G such that $E = M_1 \cup M_2 \cup \ldots \cup M_p$.

11. a) State and prove Euler's formula for a connected plane graph.

    b) If G is a simple planar graph with $\upsilon \ge 3$, prove that $\varepsilon \le 3\upsilon - 6$.

    c) Deduce that K_5 is nonplanar.

12. a) If two bridges overlap prove that either they are skew or else they are equivalent 3-bridges.

    b) Prove that every planar graph is 5-vertex colourable.

Unit – III

13. a) Prove that every digraph contains a directed path of length $\chi - 1$ and deduce that every tournament has a directed Hamilton path.

    b) Prove that every loopless digraph D has an independent set S such that each vertex of D not in S is reachable from a vertex in S by a directed path of length at most two. Further deduce that a tournament contains a vertex from which every other vertex is reachable by a directed path of length of most two.

14. a) Prove that each vertex of a disconnected tournament D with $\nu \ge 3$ is contained in a directed k-cycle, $3 \le k \le \upsilon$.

    b) If D is a strict digraph with min $\{\delta^-, \delta^+\} \ge \frac{\upsilon}{2} > 1$, prove that D contains a directed Hamilton cycle.

15. a) Define a minimum cut in a network N. Let f be a flow and K be a cut such that val f = cap K. Prove that f is a maximum flow and K is a minimum cut.

    b) Let x and y be two vertices of a digraph D such that x is not joined to y. Prove that the maximum number of internally disjoint directed (x, y) paths in D is equal to the minimum number of vertices whose deletion destroys all directed (x, y) paths in D. Also obtain the undirected version of this statement.

(4×12=48)