K21P 0561

Reg. No. : ....

Name : ....

First Semester M.Sc. Degree (CBSS-Reg./Suppl. (Including Mercy Chance)/Imp.) Examination, October 2020

(2017 Admission Onwards)

MATHEMATICS

MAT1C05 : Differential Equations

Time : 3 Hours Max. Marks : 80

Instructions : *Answer **any four** questions from Part – **A. Each** question carries **4** marks. Answer **any four** questions from Part – **B** without omitting any Unit. **Each** question carries **16** marks.*

PART – A

1. Find the indicial equation of the differential equation $x^3y'' + (\cos 2x - 1)y' + 2xy = 0$.

2. Define F(a, b, c, x) and show that $F'(a, b, c, x) = \frac{ab}{c} F(a + 1, b + 1, c + 1, x)$.

3. Define the Bessel function $J_p(x)$ and show that $\frac{d}{dx}\left[x^p J_p(x)\right] = x^p J_{p-1}(x)$.

4. Find two solutions of the system $\frac{dx}{dt} = x + y,\ \frac{dy}{dt} = 4x - 2y$.

5. If y(x) is a nontrivial solution of the differential equation $y'' + q(x)y = 0$ on [a, b] then prove that y(x) has at most a finite number of zeros in [a, b].

6. Let f(x, y) and $\frac{\partial f}{\partial y}$ be continuous functions on a closed rectangle R with sides parallel to the axes. Prove that f(x, y) satisfies the Lipschitz condition on R.

PART – B

Unit – I

7\. a) Find a solution as a power series for the initial value problem $y' = x - y$, $y(0) = 0$. Express the solution in terms of familiar functions. Also verify your solution by directly solving the initial value problem.

b) Find the general solution $y = \Sigma a_n x^n$ in the form $y = a_0 y_1(x) + a_1 y_2(x)$ where $y_1(x)$ and $y_2(x)$ are power series for the differential equation $y'' + xy' + y = 0$.

8\. a) Verify that origin is a regular singular point and calculate two independent Frobenius series solutions of the equation
$2x^2y'' + x(2x + 1) y - y = 0$

b) Find two independent Frobenius series solutions of $x^2y'' - x^2y' + (x^2 - 2) y = 0$.

9\. a) Define Gauss hypergeometric equation and obtain the hypergeometric series as a solution of this equation.

b) Find the nature of the point at infinity for the Gauss hypergeometric equation.

Unit – II

10\. a) Using $(1 - 2xt + t^2)^{-1/2} = \sum_{n=0}^{\infty} P_n(x)\, t^n$, show that

i) $P_n(1) = 1$ and $P_n(-1) = (-1)^n$

ii) $P_{2n+1}(0) = 0$ and $P_{2n}(0) = (-1)^n \dfrac{1.3.....(2n-1)}{2^n\, n!}$

b) Prove the orthogonality property of Legendre polynomials.

$$\int_{-1}^{1} P_m(x)\, P_n(x)\, dx = \begin{cases} 0 & \text{if } m \neq n \\ \dfrac{2}{2n+1} & \text{if } m = n \end{cases}$$

11\. a) Prove that $J_{-1/2}(x) = \sqrt{\dfrac{2}{\pi x}} \cos x$.

b) Prove the orthogonal property of Bessel functions :

$$\int_{0}^{1} x\, J_P(\lambda_m x)\, J_P(\lambda_n x)\, dx = \begin{cases} 0 & \text{if } m \neq n \\ \dfrac{1}{2} J_{P+1}(\lambda_n)^2 & \text{if } m = n \end{cases}$$

12\. a) If two solutions $x = x_1(t)$, $y = y_1(t)$ and $x = x_2(t)$, $y = y_2(t)$ of the system $\dfrac{dx}{dt} = a_1(t)x + b_1(t)y$, $\dfrac{dy}{dt} = a_2(t)x + b_2(t)y$ are linearly independent on $[a, b]$, then prove that $x = c_1x_1(t) + c_2x_2(t)$, $y = c_1y_1(t) + c_2y_2(t)$ is the general solution of the system on $[a, b]$ for any constants c_1 and c_2.

b) Find the general solution of the system

$\dfrac{dx}{dt} = -3x + 4y$, $\dfrac{dy}{dt} = -2x + 3y$.

Unit – III

13\. a) Let $u(x)$ be any nontrivial solution of $u'' + q(x)\, u = 0$, where $q(x) > 0$ for all $x > 0$. If $\int_{1}^{\infty} q(x)\, dx = \infty$, then prove that $u(x)$ has infinitely many zeros on the positive x-axis.

b) State and prove the Sturm comparison theorem.

14\. a) Let $f(x, y)$ be a continuous function that satisfies a Lipschitz condition on a strip defined by $a \leq x \leq b$ and $-\infty < y < \infty$. If (x_0, y_0) is any interior point of the strip, prove that the initial value problem $y' = f(x, y)$, $y(x_0) = y_0$ has a unique solution on the interval $a \leq x \leq b$.

b) For what points (x_0, y_0) does the initial value problem $y' = |y|$, $y(x_0) = y_0$ has a unique solution on some interval $|x - x_0| \leq h$?

15\. a) Find the exact solution of the initial value problem $y' = x + y$, $y(0) = 1$. Starting with $y_0(x) = 1$, calculate $y_2(x)$, $y_3(x)$ and $y_4(x)$.

b) Solve the following initial value problem (system) by Picard's method.

$\dfrac{dy}{dx} = z$, $y(0) = 1$

$\dfrac{dz}{dx} = -y$, $z(0) = 0$
