K21P 0557

Reg. No. : ....

Name : ....

First Semester M.Sc. Degree (CBSS-Reg./Suppl. (Including Mercy Chance) /Imp.) Examination, October 2020
(2017 Admission Onwards)
MATHEMATICS
MAT 1C01 – Basic Abstract Algebra

Time : 3 Hours Max. Marks : 80

PART – A

Answer **four** questions from this Part. **Each** question carries **4** marks.

1. Define normal series. Give an example.

2. Show that every group of order 45 has a normal subgroup of order 9.

3. If R is a ring with unity 1, then prove that the map $\phi : \mathbb{Z} \to R$ given by $\phi(n) = n.1$ is a homomorphism.

4. Find the sum and the product of the polynomials $f(x) = 2x^2 - 4x + 2$ and $g(x) = 4x - 5$ in $\mathbb{Z}_5[x]$.

5. Write all abelian groups up to isomorphism of order 24.

6. Define isomorphic normal series. Give an example.

PART – B

Answer **any four** questions from Part – **B**. Without omitting **any** Unit. **Each** question carries **16** marks.

Unit – I

7. a) Prove that the group $\mathbb{Z}_m \times \mathbb{Z}_n$ is cyclic and is isomorphic to $\mathbb{Z}_{mn}$ if and only if m and n are relatively prime.

   b) Let G be an abelian group. Let H be the subset consisting of the identity e together with all elements of G of order 2. Show that H is a subgroup of G.

P.T.O.

8. a) Let X be a G-set. Then prove that $|Gx| = (G : G_x)$

   b) Show that there are no simple groups of order 255.

9. a) If H and K are subgroups of a group G, then prove that $|HK| = \frac{(|H|)(|K|)}{|H \cap K|}$.

   b) Show that no group of order 30 is simple.

Unit – II

10. a) Prove that any two fields of quotients of an integral domain D are isomorphic.

    b) Describe the field of quotients of the integral subdomain $D = \{n + mi \mid n, m \in \mathbb{Z}\}$ of $\mathbb{C}$.

11. a) If N is a normal sub group of G and H is any subgroup of G, then prove that $H \vee N = HN = NH$. Also, if H is normal subgroup of G, then prove that HN is a normal subgroup of G.

    b) Show that the center of a group G is a normal subgroup of G. Find the ascending central series of D_4.

12. a) Prove that every finitely generated abelian group is isomorphic to a group of the form $\mathbb{Z}_{m_1} \times \mathbb{Z}_{m_2} \times \ldots \times \mathbb{Z}_{m_r} \times \mathbb{Z} \times \ldots \times \mathbb{Z}$ where m_i divides m_{i+1} for i = 1, ..., r – 1.

    b) Show that a free abelian group contains no nonzero elements of finite order.

Unit – III

13. a) State and prove division algorithm for F[x].

    b) Show that the polynomial $f(x) = x^4 - 2x^2 + 8x + 1$ is irreducible over $\mathbb{Q}$.

14. a) Let R be a commutative ring with unity. Then prove that if M is a maximal ideal of R if and only if R/M is a field.

    b) Show that a factor ring of a field is either the trivial ring of one element or is isomorphic to the field.

15. a) Is $x^3 + 2x + 3$ an irreducible polynomial of $\mathbb{Z}_5[x]$? Why ? Express it as product of irreducible factors.

    b) Prove that an ideal $\langle p(x) \rangle \neq \{0\}$ of F [x] is maximal if and only if p(x) is irreducible.
