K20U 3367

Reg. No. : ....

Name : ....

First Semester B.Sc. Hon's (Mathematics) Degree (Reg./Supple./Improve.) Examination, November 2020
(2016 Admission Onwards)
BHM104 : MATRICES AND PROBABILITY THEORY

Time : 3 Hours Max. Marks : 60

SECTION – A

Answer **any 4** questions out of 5 questions. **Each** question carries **1** mark. **(4×1=4)**

1. State the distributive law of dot product.
2. Find the rank of $\begin{bmatrix} 2 & 3 \\ 4 & 6 \end{bmatrix}$
3. State Cauchy-Schwartz inequality.
4. What do you mean by Random variables ?
5. Define moment generating functions of a random variable.

SECTION – B

Answer **any 6** questions out of 9 questions. **Each** question carries **2** marks. **(6×2=12)**

6. Find the angle θ between the following vectors x and y : x = [– 4, 3], y = [6, –1].
7. What do you mean by reduced row echelon form of a matrix ? Write an example.
8. Define eigenvalue of a square matrix. Find the eigenvalue of $\begin{bmatrix} 2 & 3 \\ 5 & 4 \end{bmatrix}$.
9. What do you mean by consistent solution of system of linear equations ?
10. Find the four minors of $\begin{bmatrix} 1 & 5 & 1 \\ 2 & 1 & -7 \\ -1 & 2 & 6 \end{bmatrix}$.

P.T.O.

11. If X is a random variable, prove that $var[x] = E[X^2] - [E[X]]^2$.

12. Find the cumulative distribution function for the probability function of a random variable X, $f(x) = \begin{cases} 6(x - x^2) & 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$.

13. Let X has probability distribution function $f(x) = \begin{cases} \frac{2}{25}x & 0 \le x \le 5 \\ 0 & \text{otherwise} \end{cases}$. Find $P(1 < x < 3)$.

14. If X is a random variable defined by the probability density function $f(x) = \begin{cases} 3x^2 & 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$. Find $E(3X - 2)$.

SECTION – C

Answer **any 8** questions out of 12 questions. **Each** question carries **4** marks. **(8×4=32)**

15. Using Gauss-Jordan Row reduced method to solve the following system of linear equations

$$x - 2y + z - w = 0$$
$$x + y - 2z - 3w = 0$$
$$4x + y - 5z + 8w = 0$$
$$5x - 7y + 2z - w = 0.$$

16. Suppose A and B are row space equivalent matrices. Prove that row space of A equals row space of B.

17. Find the inverse of $\begin{bmatrix} 1 & -4 & 1 \\ 1 & 1 & -2 \\ -1 & 1 & 1 \end{bmatrix}$.

18. Prove that $n \times n$ matrix is non-singular iff $|A| \neq 0$.

19. Find the rank of $\begin{bmatrix} 0 & 1 & 2 & -2 \\ 4 & 0 & 2 & 6 \\ 2 & 1 & 3 & 1 \end{bmatrix}$.

20. Briefly explain the method of diagonalization of an $n \times n$ matrix.

21. The number of components manufactured in a factory during one month period is a random variable with mean 600 and variance 100. What is the probability that the production will be between 500 and 700 over a month ?

22. If X and Y are random variables then show that
    i) $E(X + Y) = E(X) + E(Y)$
    ii) $E(XY) = E(X)E(Y)$, if X and Y are independent random variables.

23. State and prove Cauchy-Schwartz inequality.

24. Find the first four moments of the data 4, 5, 6, 1, 4 about mean.

25. A coin is tossed until a head appears. What is the expectation of the number of tosses required ?

26. The probability density function of a random variable X is given by $f(x) = \begin{cases} kx(2-x) & 0 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$. Find k, β_1 and β_2.

SECTION – D

Answer **any 2** questions out of 4 questions. **Each** question carries **6** marks. **(2×6=12)**

27. Using Gauss-Elimination method to solve the following system of linear equations. Indicate whether the system is consistent or inconsistent. Give complete solution.

$$x + 2y - 5z = -9$$
$$3x - y + 2z = 5$$
$$2x + 3y - z = 3$$
$$4x - 5y + z = -3.$$

28. Diagonalize the matrix $\begin{bmatrix} -4 & 8 & -12 \\ 6 & -6 & 12 \\ 6 & -8 & 14 \end{bmatrix}$.

29. Briefly explain Skewness and Kurtosis.

30. Two random variables X and Y have the disjoint probability density function $f(x, y) = Ae^{-(2x+y)}$, $x, y \ge 0$.
    i) Evaluate A.
    ii) Find marginalised probability density functions.

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