Reg. No. : ....

Name : ....

**First Semester B.Sc. Hon's (Mathematics) Degree (Reg./Supple./Improve.)
Examination, November 2020**
(2016 Admission Onwards)
BHM 102 : FOUNDATIONS OF MATHEMATICS

Time : 3 Hours

Max. Marks : 60

SECTION – A

Answer **any 4** questions out of 5 questions. **Each** question carries **1** mark.   **(4×1=4)**

1. Define cartesian product of two sets.

2. What is the locus of the equation $x^2/a^2 + y^2/b^2 + z^2/c^2 = -1$?

3. Give an example of a surjective function.

4. Define a countable set.

5. State least upper bound property for a set.

SECTION – B

Answer **any 6** questions out of 9 questions. **Each** question carries **2** marks. **(6×2=12)**

6. Let $f : A \rightarrow B$ be a surjective function. Let us define a relation on A by setting $a_0 \sim a_1$, if $f(a_0) = f(a_1)$. Show that this is an equivalence relation.

7. Define a polar plane.

8. Distinguish between the conjunction and disjunction of statements p and q.

9. Define dictionary order relation.

10. Find the radius and centre of the sphere $x^2 + y^2 + z^2 - 2x + 4y - 6z = 2$.

11. Define converse of a statement. Give an example.

12. Consider the function $f : R \to R$ given by $f(x) = 3x^2 + 2$. Find $f^{-1}(f([0, 1]))$ and $f(f^{-1}([0, 5]))$.

13. Find the equation of the right circular cylinder of radius 2 whose axis is the line
$$\frac{x-1}{2} = \frac{y-2}{2} = \frac{z-2}{2}.$$

14. Define the bijective function and give an example of a bijective function.

SECTION – C

Answer **any 8** questions out of 12 questions. **Each** question carries 4 marks. **(8×4=32)**

15. Show that countable union of countable sets is countable.

16. Show that two equivalence classes E and E′ are either disjoint or equal.

17. Find the least upper bound for the following sets.

a) $\left\{ 1 - \dfrac{(-1)^n}{n} : n \in N \right\}$

b) $\left\{ \dfrac{-1}{2n} : n \in N \right\}$

18. Let $a \geq 0$ be a real number. Show that if for every $\epsilon > 0$ we have $0 \leq a \leq \epsilon$, then $a = 0$.

19. Show that the square of an odd integer is an odd integer.

20. Given $f : R \to R$ by $f(x) = \cos x$ and $g : R \to R$ by $g(x) = 3x^2$. Show that $f \circ g \neq g \circ f$.

21. Show that if m, n are natural numbers such that $m+n \geq 20$, then either $m \geq 0$ or $n \geq 10$.

22. Find the equation of the sphere which touches the sphere $x^2 + y^2 + z^2 - x + 3y + 2z - 3 = 0$ at the point $(1, 1, -1)$ and passes through the origin.

23. Prove that the plane $ax + by + cz = 0$ cuts the cone $yz + zx + xy = 0$ in perpendicular lines if $1/a + 1/b + 1/c = 0$.

24. Show that the sum of the squares of the projections of three semi-conjugate diameters on any line or plane is constant.

25. Show that through any given point six normals can be drawn to a central conicoid.

26. The set Q_+ of positive rationals numbers is countably infinite.

SECTION – D

Answer **any 2** questions out of 4 questions. **Each** question carries 6 marks. **(2×6=12)**

27. Let A be a set. Show that there is no injective map $f : P(A) \to A$ and there is no surjective map $g : A \to P(A)$.

28. Find the equation of the cone with vertex $(5, 4, 3)$ and $3x^2 + 2y^2 = 6$, $y + z = 0$ as base.

29. Show that the plane $2x - 4y - z + 3 = 0$ touches the paraboloid $x^2 - 2y^2 = 3z$. Find also the co-ordinates of the point contact.

30. What are the equations relating spherical co-ordinates of a point to cartesian and cylindrical co-ordinates ? Also find spherical co-ordinate equation for the sphere $x^2 + y^2 + (z - 1)^2 = 1$.