**K20P 0339**

Reg. No. : ..............................

Name : ..................................

**II Semester M.Sc. Degree (CBSS – Suppl.) Examination, April 2020**
**(2014 – 2016 Admissions)**
**MATHEMATICS**
**MAT2C06 : Advanced Abstract Algebra**

Time : 3 Hours Max. Marks : 60

PART – A

Answer **four** questions from this Part. **Each** question carries **3** marks.

1. Prove that every PID is a UFD.
2. Using Euclidean algorithm, find the gcd of 22,471 and 3,266 on Z.
3. Prove that Z[i] is an integral domain.
4. Prove that $\sqrt{1+\sqrt{3}}$ is algebraic of degree 4 over Q.
5. Prove that complex zeros of polynomials with real coefficients occur in conjugate pairs.
6. Show that the polynomial $x^4 - 5x^2 + 6$ splits in the field $Q[\sqrt{2}, \sqrt{3}]$. **(4×3=12)**

PART – B

Answer **4** questions from this Part without omitting **any** Unit. **Each** question carries **12** marks.

**Unit – I**

7. a) Prove that <p> in a PID is maximal if and only if p is an irreducible element.

   b) Prove that if D is a UFD, then a product of two primitive polynomials in D[x] is again primitive.

8. a) If D is a UFD then prove that D[x] is also UFD.

   b) Prove that $2x - 10$ is irreducible in Z[x] and in $Z_{11}[x]$. Express the polynomial $18x^2 - 12x + 48$ in the UFD in Z[x] as the product of its constant with primitive polynomial.

P.T.O.

9. a) If D is an integral domain with a multiplicative norm N, then prove that $N(1) = 1$ and $|N(u)| = 1$ for every unit u in D.

   b) Find all units in the set of Gaussian integers Z[i].

**Unit – II**

10. a) Prove that a field F is algebraically closed if and only if every non-constant polynomial in F[x] factors in F[x] into linear factors.

    b) Prove that there exists an angle that cannot be trisected with a straight edge and a compass.

11. a) Prove that the multiplicative group $(F^*, .)$ of nonzero elements of a finite field F is cyclic.

    b) Let p be a prime number and let $n \in Z^+$. Prove that any two fields of $p^n$ are isomorphic.

12. a) Prove that a finite field $GF(p^n)$ of $p^n$ elements exists for every prime power $p^n$.

    b) Prove that doubling the cube is impossible.

**Unit – III**

13. a) Prove that the field C of complex numbers is an algebraically closed field.

    b) Prove that an element $\alpha$ of an extension field E of a field F is algebraic over F if and only if $\alpha$ is a zero of some polynomial.

14. a) Prove that every finite field is perfect.

    b) If $F \leq E \leq K$, where K is a finite extension field of the field F, then prove that $\{K : F\} = \{K : E\} \{E : F\}$.

15. a) Prove that a finite extension of a field of characteristic zero is a simple extension.

    b) If E is a extension of F, then prove that E is separable over F if and only if each $\alpha$ in E is separable over F. **(4×12=48)**

---