K20P 0348

Reg. No. : ....

Name : ....

II Semester M.Sc. Degree (CBSS – Reg./Suppl./Imp.) Examination, April 2020
(2017 Admission Onwards)
MATHEMATICS
MAT2C08 – Advanced Topology

Time : 3 Hours Max. Marks : 80

PART – A

Answer **any four** questions from this part. **Each** question carries **4** marks : **(4×4=16)**

1. Give an example, with proper reasoning, of a bounded metric space that is not compact.
2. Is a continuous function from a compact metric space to a metric space always uniformly continuous ? Justify your answer.
3. Show that regularity is a topological property.
4. Is the topological space (X, T) normal, where X = {1, 2, 3, 4} and T = {ϕ, {1}, {1, 2}, {1, 2, 3}, X} ? Justify your answer.
5. Show that a T_1 space, which can be imbedded as a subspace of I^∞, is a separable metric space.
6. Let (X_n, d_n) be a metric space for each $n \in \mathbb{N}$ and let $X = \Pi_{n\in\mathbb{N}} X_n$. Prove that $d(x, y) = \sum_{n=1}^{\infty} \frac{d_n(x_n, y_n)}{2^n}$ for x, y∈X is a metric on X.

PART – B

Answer **any four** questions from this part without omitting **any** unit.
Each question carries **16** marks : **(4×16=64)**

Unit – I

7. a) Prove that every open cover of a metric space with the Bolzano-Weierstrass property has a Lebesgue number.
   b) Prove that a metric space is compact if and only if it is complete and totally bounded.

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8. a) Show that the product of two compact spaces is compact.
   b) Show that compactness is a topological property.
   c) Give an example, with proper reasoning, of a compact set that is not closed.
9. a) Prove that every closed subspace of a locally compact Hausdorff space is locally compact.
   b) Let X be a locally compact space. If there is an open continuous function from X onto Y, then show that Y is locally compact.
   c) Give an example, with proper reasoning of a compact set that is not sequentially compact.

Unit – II

10. a) Give an example, with proper reasoning, of a T_1 space that is not T_2.
    b) Let X be a topological space and Y a Hausdorff space. If $f : X \to Y$ is continuous, then prove that $\{(x_1, x_2) \in X \times X : f(x_1) = f(x_2)\}$ is a closed set.
    c) Prove that a T_1 space is regular if and only if for each $p \in X$ and each neighbourhood U of p, there is a neighbourhood V of p such that $\overline{V} \subseteq U$.
11. a) Let $\{(X_\alpha, T_\alpha) : \alpha \in \wedge\}$: be a family of topological spaces with $X = \Pi_{\alpha \in \wedge} X_\alpha$. Prove that (X, T) is regular if and only if (X_α, T_α) is regular for each $\alpha \in \wedge$.
    b) Let $(X, \leq)$ be a well-ordered set and let T denote the order topology on X. Prove that (X, T) is a normal space.
12. a) Prove that a T_1 space is completely normal if and only if each of its subspace is normal.
    b) Prove that every regular Lindelof space is normal.

Unit – III

13. a) State and prove Urysohn's Lemma.
    b) Prove that the set of dyadic numbers in I is dense in I.
14. a) State and prove Tychonoff theorem.
    b) Prove that, if (X, T) is a T_1, regular and second countable space, then X can be imbedded as a subspace of I^ω.
    c) Show that the space I^ω is mertrizable.
15. a) For two spaces (X, T). (Y, U), show that the relation defined by $f \simeq g$ if f is homotopic to g is an equivalence relation on C(X, Y).
    b) Let (X, T) be a topological space and $x_0 \in X$. Prove that the operation $\circ$ defined on $\pi_1(X, x_0)$ by $[\alpha] \circ [\beta] = [\alpha * \beta]$ is associative.
