Reg. No. : ....

Name : ....

II Semester M.Sc. Degree (CBSS-Reg./Suppl./Imp.) Examination, April 2020

(2017 Admission Onwards)

MATHEMATICS

MAT2C10 : Partial Differential Equations and Integral Equations

Time : 3 Hours Max. Marks : 80

PART – A

Answer **any four** questions from this Part. **Each** question carries **4** marks. **(4×4=16)**

1. Write the partial differential equation of $z = F\left(\frac{xy}{z}\right)$ by eliminating the arbitrary function F.
2. Find the general integral of the partial differential equation yzp + xzq = xy.
3. State Green's theorem and write the conditions of the functions involved.
4. State Cauchy problem and give an example.
5. Define separable Kernel. Write an example of a Fredholm integral equation involving separable kernel.
6. Convert the differential equation $y'' + 2y = 0$ with the conditions $y(0) = 0, y'(0) = 0$ to an integral equation.

PART – B

Answer **any four** questions from this Part, without omitting any Unit. **Each** question carries **16** marks. **(4×16=64)**

Unit – 1

7. a) Find the general integral of the partial differential equation $z_t + zz_x = 0$. Also verify that the obtained solution is unbounded as t tends to 1.

   b) Solve the partial differential equation $z^2 + zu_x - u_x^2 - u_y^2 = 0$ using Jacobi method.

8. a) Prove that there exist an integrating factor for a Pfaffian differential equation in two variables.

   b) Verify that the Pfaffian differential equation $yzdx + xzdy + xydz = 0$ is exact. Also find its integral.

9. a) Define compatible system of first order partial differential equations in a domain. Also write the condition that this compatible system is integrable.

   b) Prove that the system of equations $f = p^2 + q^2 - 1 = 0$; $g = (p^2 + q^2)x - pz = 0$ are compatible and find the one-parameter family of common solutions.

Unit – 2

10. a) Write the general form of a second order semi-linear partial differential equation. Based on different conditions, give one example of Hyperbolic, Parabolic and Elliptic type of a second order semi-linear partial differential equation.

    b) Reduce the equation $y^2u_{xx} - 2xyu_{xy} + x^2u_{yy} = \frac{y^2}{x}u_x + \frac{x^2}{y}u_y$ to a canonical form and solve it.

11. a) Find the d' Alembert's solution of the one-dimensional wave equation

    $\frac{\partial^2 y}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2}$ with initial conditions $y(x, 0) = f(x)$, $y_t(x, 0) = g(x)$, $-\infty < x < \infty$, $t > 0$.

    b) Write the characteristic curves of the above one-dimensional wave equation.

12. a) Prove that the solution of Neumann problem is unique up to the addition of a constant.

    b) Solve the heat conduction equation $u_{tt} - c^2u_{xx} = F(x, t)$, $0 < x < l$, $t > 0$ satisfying the initial conditions $u(x, 0) = f(x)$, $0 < x < l$, $u_t(x, 0) = g(x)$, $0 < x < l$, $u(0, t) = u(l, t) = 0$, $t > 0$ by making use of Duhamel's principle. Also write the uniqueness condition for the obtained solution.

Unit – 3

13. Transform the differential equation $\frac{d^2y}{dx^2} + xy = 1$ with the condition $y(0) = 0$, $y(l) = 1$ to a Fredholm integral equation using Green's function.

14. a) Solve the Fredholm integral equation $y(x) = \lambda\int_0^1 (1 - 3x\xi)y(\xi)d\xi + F(x)$ in the following two cases.

    i) $F(x) = 0$.

    ii) $F(x) = x$.

    b) Find out the eigen values and the eigen functions in the two cases of part (a).

15. a) Using iterative method, solve the Fredholm equation of the second kind

    $$y(x) = 1 + \lambda\int_0^1 (1 - 3x\xi)y(\xi)\, d\xi .$$

    b) For what condition, the solution of part (a) is convergent ?