K19P 0349

Reg. No. : ....

Name : ....

II Semester M.Sc. Degree (Suppl.) Examination, April 2019
(2014 – 2016 Admission)
MATHEMATICS
MAT 2C 06 : Advanced Abstract Algebra

Time : 3 Hours Max. Marks : 60

PART – A

Answer **any 4** questions. **Each** question carries **3** marks.

1. Prove that the ascending chain condition holds for the ideals in $\mathbb{Z}$.
2. Give an example of an integral domain that is not a UFD. Justify your claim.
3. Prove that the field $\mathbb{Q}\left(\sqrt{2}\right)$ does not contain any cube root of 2.
4. Construct a field of 9 elements.
5. Describe the group of automorphisms of $\mathbb{Q}\left(\sqrt[4]{2}\right)$ leaving $\mathbb{Q}$ fixed.
6. Find all extensions of the automorphism $\psi_{\sqrt{2},-\sqrt{2}}$ of $\mathbb{Q}\left(\sqrt{2}\right)$ to an isomorphism mapping $\mathbb{Q}\left(\sqrt{2},\sqrt{3}\right)$ to a subfield of $\overline{\mathbb{Q}}$.

PART – B

Answer **4** questions without omitting **any** Unit. **Each** question carries **12** marks.

Unit – I

7. a) Prove that every PID is a UFD.
   b) Prove that in a PID, every ideal is contained in a maximal ideal.
8. a) Prove that every Euclidean domain is a PID.
   b) Verify whether or not the integral domain $\mathbb{Z}[x]$ is a (i) UFD (ii) PID.
9. a) Prove that an odd prime p is not an irreducible in $\mathbb{Z}[i]$ if $p \equiv 1 \pmod 4$.
   b) Find the irreducible polynomial of $\sqrt{1+\sqrt{1+\sqrt{11}}}$ over $\mathbb{Q}$. Justify your claim.

P.T.O.

Unit – II

10. a) Give an example of an algebraically closed field. Justify your claim.

 b) What are the irreducible polynomials in F[x], if F is an algebraically closed field ?

 c) Prove the impossibility of squaring a circle.

11. a) Prove that for any prime p and for any positive integer n, there exists a finite field of p^n elements.

 b) Prove the algebraic closure of $\mathbb{Z}_p$ is not a finite extension of $\mathbb{Z}_p$; for any prime p.

12. a) State and prove the conjugation isomorphism theorem.

 b) Prove that an automorphism of the field $\mathbb{R}$ carries positive numbers onto positive numbers.

Unit – III

13. a) Prove that an algebraic extension E of F is a splitting field over F if and only if every automorphism of $\bar{F}$ leaving F fixed induces an automorphism of E leaving F fixed.

 b) Compute $\{Q\ \sqrt[3]{2} : Q\}$.

14. a) Let K be a finite extension of E and E be a finite extension of F. Prove that K is separable over F if and only if K is separable over E and E is separable over F.

 b) Prove that every irreducible polynomial in F[x] has a factorization of the form a $\prod_{i=1}^{n}(x-\alpha_i)^{\gamma}$.

 c) Is there a finite extension of Q, which is not separable ? Why ?

15. a) Let K be a finite normal extension of F and E be an extension of F such that $F \leq E \leq K$. Prove or disprove :

 i) E is a normal extension of F.

 ii) K is a normal extension of E.

 b) Illustrate the main theorem of Galois theory for the normal extension $Q(\sqrt{2}, \sqrt{5})$ of Q.