Reg. No. : ....

Name : ....

K18P 0136

Second Semester M.Sc. Degree (Supplementary/Improvement) Examination, March 2018

MATHEMATICS

(2014 – 2016 Admn.)

MAT2C08 : Topology

Time : 3 Hours Max. Marks : 60

PART – A

Answer **any four** questions from this Part. **Each** question carries **3** marks. **(4×3=12)**

1. Prove that every subspace of a completely regular space is completely regular.
2. Prove that every second countable space is Lindelof.
3. For each $n \in \mathbb{N}$, let $B_n = \{2n - 1, 2n\}$. Then $B = \{B_n : n \in \mathbb{N}\}$ is a base for a topology T on $\mathbb{N}$. Show that $(\mathbb{N}, T)$ has the Bolzano-Weierstrass property, but it is not countably compact.
4. Show that the one-point compactification of R is homeomorphic to the circle S^1.
5. Define homotopy between two continuous functions $f, g : X \to Y$, where X and Y are topological spaces and give an example.
6. Let (X, T) be a topological space and $x_0 \in X$. Prove that path homotopy is an equivalence relation on $\Omega(X, x_0)$.

PART – B

Answer **any four** questions from this Part without omitting any Unit. **Each** question carries **12** marks. **(4×12=48)**

UNIT – I

7. a) Let (X, T) be a topological space. Prove that (X, T) is T_1-space if and only if $A = \cap\{U \in T : A \subseteq U\}$ for any subset A of X.

P.T.O.

b) Let $\{(X_\alpha, T_\alpha) : \alpha \in \Lambda\}$ be a family of topological spaces and let $X = \prod_{\alpha \in \Lambda} X_\alpha$.

Prove that (X, T) is regular if and only if (X_α, T_α) is regular for each $\alpha \in \Lambda$.

8. a) Let (X, ≤) be a well-ordered set and let T denote the order topology on X. Prove that (X, T) is a normal space.

b) Prove that a T_1-space (X, T) is completely normal if and only if every subspace of X is normal.

9. a) Prove that every regular Lindelof space is normal.

b) Prove that a T_1-space (X, T) is normal if and only if whenever A is a closed subset of X and $f : A \to [-1, 1]$ is a continuous function then there is a continuous function $F : X \to [-1, 1]$ such that $F|_A = f$.

UNIT – II

10. a) Prove that every closed subspace of a locally compact Hausdorff space is locally compact.

b) Let (X, T) be a compact space, let (Y, d) be a compact metric space and let $F \subseteq C(X, Y)$. Prove that F is equicontinuous if and only if F is totally bounded with respect to sup metric.

11. a) Define one-point compactification. Prove that the one-point compactifications of two homeomorphic topological spaces are homeomorphic.

b) State (without proof) Alexander subbase theorem. Use it prove that the product of compact spaces is compact.

12. State and prove Urysohn's metrization theorem.

UNIT – III

13. Let (X, T) be a topological space and $x_0 \in X$. Let $\Pi_1(X, x_0)$ denote the set of all path-homotopy equivalence classes on $\Omega(X, x_0)$ and define an operation o on $\Pi_1(X, x_0)$ by $[\alpha] \circ [\beta] = [\alpha * \beta]$.

a) Prove that the operation o is well defined.

b) Prove that the operation o is associative.

14. a) Let (X, T) be a topological space and $x_0 \in X$. Prove that $(\Pi_1(X, x_0), o)$ has an identity element.

b) Define a contractible space. Let (X, T) be a contractible space and (Y, U) be a pathwise connected space. If $f, g : X \to Y$ are continuous functions, then prove that f is homotopic to g.

15. a) Let (X, T) be a pathwise connected space and let $x_0, x_1 \in X$. Prove that $\Pi_1(X, x_0)$ is isomorphic to $\Pi_1(X, x_1)$.

b) Let (X, T) and (Y, U) be topological spaces and let $x_0 \in X$ and $y_0 \in Y$. When do you say that (X, x_0) and (Y, y_0) are of the same homotopy type ? If (X, x_0) and (Y, y_0) are of the same homotopy type, then prove that $\Pi_1(X, x_0)$ is isomorphic to $\Pi_1(Y, y_0)$.