K18P 0231

Reg. No. : ....

Name : ....

Second Semester M.Sc. Degree (Regular) Examination, March 2018
MATHEMATICS (2017 Admn.)
MAT 2 C09 : Foundations of Complex Analysis

Time : 3 Hours | Max. Marks : 80

PART – A

Answer **any four** questions from this part. **Each** question carries **4** marks.

1. Evaluate $\int_{r} \frac{z^2+1}{z(z^2+4)} dz$ where $r(t) = re^{it}$, $0 \leq t \leq 2\pi$, for all possible values of r with $0 < r < 2$.

2. Show that the relation homotopy is an equivalence relation on the set of all closed rectifiable curves in a region.

3. Define :
   i) isolated singularity
   ii) removable singularity.
   Illustrate with examples.

4. Does there exist an analytic function $f : D \to D$ with $f\left(\frac{1}{2}\right) = \frac{3}{4}$ and $f'\left(\frac{1}{2}\right) = \frac{2}{3}$ (where $D = \{z : |z| < 1\}$) ? Why ?

5. Define the set $C(G, \Omega)$. Can it be empty ? Why ?

6. Show that $\prod_{n=2}^{\infty}\left(1-\frac{1}{n^2}\right) = \frac{1}{2}$. **(4×4=16)**

P.T.O.

PART – B

Answer **any four** questions from this part without omitting **any** unit. **Each** question carries **16** marks.

Unit – I

7. a) Let G be a connected open set and let $f : G \to \mathbb{C}$ be an analytic function. Prove that $f \equiv 0$ if and only if the set $\{z \in G : f(z) = 0\}$ has a limit point in G.

   b) State and prove the maximum modulus theorem.

8. a) Define the winding number and prove that it is an integer.

   b) State and prove the first version of Cauchy's integral formula.

9. a) Let G be a region and let $f : G \to \mathbb{C}$ be a continuous function such that $\int_T f = 0$ for every path T in G. Prove that f is analytic in G.

   b) Let G be an open set and let $f : G \to \mathbb{C}$ be a differentiable function. Prove that f is analytic on G.

Unit – II

10. a) State the theorem (no proof) on Laurent series development. Use the Laurent expansion to classify the isolated singularity at a point z = a of a function f analytic in $\{z : 0 < |z - a| < R\}$. Justify your classification.

    b) State and prove Casaroti-Weierstrass theorem.

11. a) State and prove residue theorem.

    b) Use residue theorem to show that $\int_0^\infty \frac{\sin x}{x}\,dx = \frac{\pi}{2}$.

12. a) State and prove Schwarz's lemma.

    b) If $|a| < 1$, prove that the map φ_a defined by $\varphi_a(z) = \frac{z-a}{1-\bar{a}z}$ is a one-one map of $D = \{z : |z| < 1\}$ onto itself; the inverse of φ_a is φ_{-a}. Also prove that φ_a maps ∂D onto ∂D, $\varphi_a(a) = 0$, $\varphi_a'(0) = 1 - |a|^2$ and $\varphi_a'(a) = (1 - |a|^2)^{-1}$.

Unit – III

13. a) Suppose $F \subset C(G, \Omega)$ is equicontinuous at each point of G. Prove that F is equicontinuous over each compact subset of G.

    b) State and prove Arzela-Ascoli theorem.

14. a) Define the set H(G). If $\{f_n\}$ is a sequence in H(G) and f belongs to $C(G, \mathbb{C})$ such that $f_n \to f$ then prove that f is analytic and $f_n^{(k)} \to f^{(k)}$ for each integer $k \geq 1$.

    b) Prove that a family F in H(G) is normal if and only if F is locally bounded.

15. a) State (no proof) the Weierstrass factorization theorem.

    b) Let G be a region and let $\{a_j\}$ be a sequence of distinct point in G with no limit point in G; and let $\{m_j\}$ be a sequence of integers. Then prove that there is an analytic function f defined on G whose only zeros are at the points a_j and further that a_j is a zero of multiplicity m_j. **(4×16=64)**

____