K16P 0421

Reg. No. : ....

Name : ....

Second Semester M.Sc. Degree (Reg./Supple./Impr.)
Examination, March 2016
(2014 Admn. Onwards)
MATHEMATICS
MAT 2C06 : Advanced Abstract Algebra

Time : 3 Hours Max. Marks : 60

PART – A

Answer **any 4** questions. **Each** question carries **3** marks.

1. Give an example to show that an integral domain can contain irreducibles that are not primes.

2. Find a g.c.d. of 16 + 7 i and 10 – 5i in Z [i].

3. Describe explicitly the smallest field containing Q and the real fifth root of 2.

4. Discuss the constructibility of the real numbers $\sqrt[4]{2}$ and $\sqrt[5]{2}$.

5. Find the irreducible polynomial of $\sqrt{1+\sqrt{7}}$ over Q.

6. Give an example of an extension of Q, which is

   i) a normal extensioin

   ii) not a normal extension.

P.T.O.

PART – B

Answer **any 4** questions without omitting any Unit. **Each** question carries **12** Marks.

UNIT – I

7. a) Prove that an ideal <p> in a PID is maximal if and only if p is an irreducible.

   b) Give an example of an integral domain for which the ascending chain condition holds but the descending chain condition does not hold.

8. a) Prove that every Euclidean domain is a PID.

   b) If F is a field, prove that F [x] is a Euclidean domain by giving a Euclidean valuation on F [x].

9. a) Prove that if an odd prime p is irreducible in $\mathbb{Z}$ [i], then $p \equiv 3 \pmod 4$.

   b) State Kronecker's theorem. Find an extension of $\mathbb{Q}$ containing a zero of $x^4 - 5x^2 + 6$.

UNIT – II

10. a) Let E be an algebraic extension of F. Prove that there exist a finite number of elements $\alpha_1, \ldots, \alpha_n$ in E such that $E = F(\alpha_1, \ldots, \alpha_n)$ if and only if E is a finite extension of F.

    b) Is the real number $\sqrt[3]{2}$ belongs to the real field $\mathbb{Q}\left(\sqrt[4]{2}\right)$? Justify your claim.

11. a) Prove that every element in a finite field of characteristic p, is a zero of the polynomial $x^{p^n} - x$ in $\mathbb{Z}_p$ [x] for some positive integer n.

    b) Prove that if $\alpha > 0$ is a constructible real number then $\sqrt{\alpha}$ is also constructible.

12. a) Prove that if F is a finite field of characteristic p, then the fixed field of the Frobenius automorphism of F is isomorphic to $\mathbb{Z}_p$.

    b) Find all the isomorphisms of the field $\mathbb{Q}\left(\sqrt{2}, \sqrt[3]{2}\right)$ onto subfields of $\bar{\mathbb{Q}}$. Which of them are automorphisms ?

UNIT – III

13. a) Let E be a finite extension of a field F, σ be an isomorphism of F onto a field F' and let $\overline{F'}$ be an algebraic closure of F'. Prove that the number of extensions of σ to an isomorphism of E onto a subfield of $\overline{F'}$ is finite and independent of σ, F' and $\overline{F'}$.

    b) Prove that the field $\mathbb{Q}\left(\sqrt[4]{2}\right)$ is not a splitting field over $\mathbb{Q}$.

14. a) If E is a finite extension of F, then prove that {E : F} divides [E : F].

    b) Prove that every field of characteristic zero is perfect.

15. a) Prove that if K is a finite extension of a finite field F, then

       i) K is a normal extension of F and

       ii) The Galois group is cyclic.

    b) Illustrate the main theorem of Galois theory for the extension GF (p^{12}) of $\mathbb{Z}_p$.

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