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MINIMUM K16P 0423

Reg. No.	:
Name :	

II Semester M.Sc. Degree (Regular/Supplementary/Improvement)

Examination, March 2016

(2014 Admn. Onwards)

MATHEMATICS

MAT 2C08 : Topology

Time: 3 Hours Max. Marks: 60

Instructions: Answer four questions from Part – A. Each question carries three marks. Answer four questions from Part – B without omitting any Unit. Each question carries 12 marks.

## PART-A

- 1. Let  $x = \{(x, y) \in IR \times IR : y = 0 \text{ or } y = 1\}$  and let z be the subspace topology on x. For each  $a(\neq 0) \in IR$ , let  $D_a = \{(a, 0), (a, 1)\}$  and let  $\mathscr{D} = \{Da = a \in IR\} \cup \{(0, 0)\} \cup \{0, 1\}$  If  $\mu$  is the quotient topology on  $\mathscr{D}$  induced by the natural map  $p: X \to \mathscr{D}$  then prove that p is open.
- Let (X, τ) be a T<sub>1</sub> space and let (Y, μ) be a topological space and let f be a closed map from X onto Y. Prove that (Y, μ) is a T<sub>1</sub>- space.
- Let (X,τ) be a topological space and let (Y, μ) be its one point compactification.
   Prove that (Y, μ) is Hausdorff if and only if (X, μ) is Hausdorff and locally compact.
- Show by an example that there exists a topological space X such that at least one open of it is finitely inadequate.
- When do we say that two functions f and g from the topological space X into the topological Y are homotopic. Illustrate it with an example.
- 6. Let X be a convex subset of IR. Prove that upto isomorphism the fundamental group of X is independent of the base point.

### PART-B

#### Unit - I

- a) Let (X, τ) be a topological space with a dense subset D and a closed relatively discrete subset c such that P(D) < c. Then prove that (X, τ) is not normal.</li>
  - Define Moore plane and deduce the from (a) that the Moore plane is not normal.
- a) Let (X ≤) be a well ordered set, and τ the order topology on X. Prove that (X,τ) is a normal space.
  - b) Prove that the real line IR with lower limit topology is Lindelöf.
- a) Let (X,τ) be a topological space. Prove that the following statements are equivalent.
  - i) (X,τ) is a τ<sub>1</sub> space.
  - ii) For each x∈X, {x} is closed.
  - iii) If A is any subset of X then  $A = \bigcap \{ \cup \in \tau : A \le \cup \}$ .
  - b) Prove that every regular Lindeto space is normal.

# Unit - II

- 10. a) Let  $(\Omega, \leq)$  be an uncountable well-ordered set with a maximal element w, having the properly that if  $x \in \Omega$  and  $x \neq w_1$  then  $\{y \in \Omega : y \leq x\}$  is countable. Let  $\tau$  be the order topology on  $\Omega$ , and let  $\Omega_0 = \Omega \{w_1\}$ . Then prove that  $(\Omega_0, \tau_{\Omega_0})$  is countably compact but not compact.
  - b) Define a k-space and give an example of it.
  - c) Prove that the quotient space of a locally compact space is a k-space.
- 11. a) Let (X, τ) be a completely regular space, let (Y, μ) be a compact Hausdorff space and let h: X → Y be a continuous function. Then prove that there is a continuous function H: 𝔻(X) → Y such that H₀e = h.

- b) Let (X,τ) be a completely regular space, let (k, h) be a compactification of X with the properly that if (Y, k) is any compactification of X. Such that each continuous function f: X → Y can be extended to a continuous in F: K → Y where F₀h = &. Prove that K is homeomorphic to B(X).
- 12. a) Let (X,τ) be a compact space, let (y, d) be a compact metric space, and let F c c(X, Y). Then prove that F is equicontinuous if and only if F is totally bounded with respect to I. Where I is the supremum metric of C(X, Y).
  - b) Define a compactification of a locally compact Hausdorff space (X,τ).

#### Unit - III

- 13. a) Let  $(X, \tau)$  be a topological space, let  $x_0 \in X$  and let  $[\alpha]$ ,  $[\beta]$ ,  $[\gamma] \in \pi_1(X, x_0)$ . Then prove that  $([\alpha] \circ [\beta]) \circ [\gamma] = [\alpha] \circ ([\beta] \circ [\gamma])$ .
  - b) Let  $(X, \tau)$  and  $(Y, \mu)$  be topological spaces, let  $x_0 \in X$  and  $y_0 \in Y$ , and let  $h = (X, x_0) \to (Y, y_0)$  be a map. Then prove that h induces a homeomorphism  $h_* = \pi_1(X, x_0) \to \pi_1(Y, y_0)$ .
- 14. a) Let  $(X, \tau)$  be a topological space, let  $x_0 \in X$  and let  $[\alpha] \in \pi_1(X, x_0)$  then prove that there exists  $[\overline{\alpha}] \in \pi_1(X, x_0)$  such that  $[\alpha] \circ [\overline{\alpha}] = [\overline{\alpha}] \circ [\alpha] = [e]$ , where  $e: I \to X$  be the path defined by  $e(x) = x_0, x \in I$ .
  - b) Let (X, τ) and (Y, μ) be topological spaces, let x<sub>0</sub> ∈ X and y<sub>0</sub> ∈ Y and let h, k: (X, x<sub>0</sub>)→(Y, y<sub>0</sub>) be maps such that h≃V vel x<sub>0</sub>. Then prove that h<sub>\*</sub> = k<sub>\*</sub>.
- 15. a) Let (X, τ) be a topological space, let x<sub>0</sub> ∈ X and let e: I → X be the path defined by e(x) = x<sub>0</sub> for each x ∈ I then prove that
  [α] ∘ [e] = [e] ∘ [α] = [α], ∀[α] ∈ π<sub>1</sub>(X, x<sub>0</sub>).
  - b) Let  $(X, \tau)$  be a path connected space and let  $x_0, x_1 \in X$  then prove that  $\pi_1(X, x_0)$  is isomorphic to  $\pi_1(X, x_1)$ .