K16P 0424

Reg. No. : ....

Name : ....

Second Semester M.Sc. Degree (Regular/Supplementary/Improvement)

Examination, March 2016

MATHEMATICS

(2014 Admn. Onwards)

MAT 2C09 : Complex Analysis

Time : 3 Hours Max. Marks : 60

PART – A

Answer **four** questions from this Part. **Each** question carries **3** marks.

1. Prove that two harmonic conjugates of a harmonic function defined in a region differ by a constant.

2. Use partial fractions to evaluate $\int_r \frac{dz}{z^2+1}$, where $r(t) = 2e^{it}$, $0 \le t \le 2\pi$.

3. Show that $\int_0^\infty \frac{dx}{x^2+1} = \frac{\pi}{2}$.

4. Let $D = \{z : |z| < 1\}$. Does there exist a function $f : D \to D$ with $f\left(\frac{1}{2}\right) = \frac{3}{4}$ and $f'\left(\frac{1}{2}\right) = \frac{3}{4}$? Justify your answer.

5. Find the value of $\prod_{n=2}^{\infty}\left(1 - \frac{1}{n^2}\right)$.

6. Derive the functional equation satisfied by the gamma function.

P.T.O.

PART – B

Answer **any four** questions from **each** Part without omitting **any** Unit. **Each** question carries **12** marks.

Unit – I

7. a) Let G be either the whole plane $\mathbb{C}$ or come open disk. If $f : G \to \mathbb{R}$ is a harmonic function, prove that u has a harmonic conjugate.

b) Let f be analytic in B (a ; R). Prove that $f(z) = \sum_{n=0}^{\infty} a_n (z-a)^n$ for $|z-a| < R$,

where $a_n = \frac{1}{n!} f^{(n)}(a)$ and this series has radius of convergence $\geq R$.

8. a) State and prove maximum modulus theorem.

b) Define the index n (r, a) of a closed curve with respect to the point a. If r is a closed rectifiable curve in $\mathbb{C}$, then prove that n (r, a) is constant for a belonging to the component of $G = \mathbb{C} - \{r\}$. Also prove that n (r, a) = 0 for a belonging to the unbounded component of G.

9. a) State and prove Morera's theorem.

b) If G is simply connected and $f : G \to \mathbb{C}$ is analytic in G, then prove that f has a primitive in G.

Unit – II

10. a) If f has an isolated singularity then, prove that the point z = a is a removable singularity if and only if $\lim_{z \to a} (z-a) f(z) = 0$.

b) State and prove Casarotic-Weierstrass theorem.

11. a) State and prove the residue theorem.

b) Show that $\int_0^{\infty} \frac{\log x}{1+x^2} dx = 0$.

12. a) State and prove Rouche's theorem.

b) State and prove the second version of the maximum modulus theorem. Is it possible to drop the assumption of boundedness of G ? Justify.

Unit – III

13. a) With usual notations prove that C (G, Ω) is a complete metric space.

b) Define equicontinuity at a point and over a set. Suppose $F \subset C(G, \Omega)$ is equicontinuous at each point of G. Then prove that F is equicontinuous over each compact subset of G.

14. a) State and prove Montel's theorem.

b) State and prove Weierstrass factorization theorem.

15. a) Define the gamma function and derive the Gauss's formula

$$\Gamma(z) = \lim_{n \to \infty} \frac{n!\ n^z}{z(z+1)\ldots(z+n)}, \quad z \neq 0, -1, \ldots$$

b) Prove that $\left(\frac{\Gamma'(z)}{\Gamma(z)}\right)' = \frac{1}{z^2} + \sum_{n=1}^{\infty} \frac{1}{(n+z)^2}$.

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