K16P 0423

Reg. No. : ....

Name : ....

II Semester M.Sc. Degree (Regular/Supplementary/Improvement) Examination, March 2016
(2014 Admn. Onwards)
MATHEMATICS
MAT 2C08 : Topology

Time : 3 Hours Max. Marks : 60

***Instructions** : Answer **four** questions from Part – **A. Each** question carries **three** marks. Answer **four** questions from Part – **B** without omitting **any** Unit. **Each** question carries **12** marks.*

PART – A

1. Let $x = \{(x, y) \in \mathbb{R} \times \mathbb{R} : y = 0 \text{ or } y = 1\}$ and let z be the subspace topology on x. For each $a(\neq 0) \in \mathbb{R}$, let $D_a = \{(a, 0), (a, 1)\}$ and let $\mathscr{D} = \{Da = a \in \mathbb{R}\} \cup \{(0,0)\} \cup \{0, 1\}$ If μ is the quotient topology on $\mathscr{D}$ induced by the natural map $p : X \to \mathscr{D}$ then prove that p is open.

2. Let (X, τ) be a T_1 space and let (Y, μ) be a topological space and let f be a closed map from X onto Y. Prove that (Y, μ) is a T_1 - space.

3. Let (X, τ) be a topological space and let (Y, μ) be its one point compactification. Prove that (Y, μ) is Hausdorff if and only if (X, μ) is Hausdorff and locally compact.

4. Show by an example that there exists a topological space X such that at least one open of it is finitely inadequate.

5. When do we say that two functions f and g from the topological space X into the topological Y are homotopic. Illustrate it with an example.

6. Let X be a convex subset of $\mathbb{R}^n$. Prove that upto isomorphism the fundamental group of X is independent of the base point.

P.T.O.

PART – B

Unit – I

7. a) Let (X, τ) be a topological space with a dense subset D and a closed relatively discrete subset c such that $p(D) < c$. Then prove that (X, τ) is not normal.

   b) Define Moore plane and deduce the from (a) that the Moore plane is not normal.

8. a) Let $(X \leq)$ be a well ordered set, and τ the order topology on X. Prove that (X, τ) is a normal space.

   b) Prove that the real line IR with lower limit topology is Lindelöf.

9. a) Let (X, τ) be a topological space. Prove that the following statements are equivalent.

   i) (X, τ) is a τ_1 space.

   ii) For each $x \in X$, {x} is closed.

   iii) If A is any subset of X then $A = \cap\{\cup \in \tau : A \leq \cup\}$.

   b) Prove that every regular Lindeto space is normal.

Unit – II

10. a) Let $(\Omega, \leq)$ be an uncountable well-ordered set with a maximal element w, having the properly that if $x \in \Omega$ and $x \neq w_1$ then $\{y \in \Omega : y \leq x\}$ is countable. Let τ be the order topology on Ω, and let $\Omega_0 = \Omega - \{w_1\}$. Then prove that $(\Omega_0, \tau_{\Omega 0})$ is countably compact but not compact.

    b) Define a k-space and give an example of it.

    c) Prove that the quotient space of a locally compact space is a k-space.

11. a) Let (X, τ) be a completely regular space, let (Y, μ) be a compact Hausdorff space and let $h : X \to Y$ be a continuous function. Then prove that there is a continuous function $H : \mathcal{B}(X) \to Y$ such that $H_o e = h$.

    b) Let (X, τ) be a completely regular space, let (k, h) be a compactification of X with the properly that if (Y, k) is any compactification of X. Such that each continuous function $f : X \to Y$ can be extended to a continuous in $F : K \to Y$ where $F_o h = \&$. Prove that K is homeomorphic to $\mathcal{B}(X)$.

12. a) Let (X, τ) be a compact space, let (y, d) be a compact metric space, and let $F \subseteq c(X, Y)$. Then prove that F is equicontinuous if and only if F is totally bounded with respect to *l*. Where *l* is the supremum metric of $\mathbb{C}(X, Y)$.

    b) Define a compactification of a locally compact Hausdorff space (X, τ).

Unit – III

13. a) Let (X, τ) be a topological space, let $x_0 \in X$ and let $[\alpha], [\beta], [\gamma] \in \pi_1(X, x_0)$. Then prove that $([\alpha] \circ [\beta]) \circ [\gamma] = [\alpha] \circ ([\beta] \circ [\gamma])$.

    b) Let (X, τ) and (Y, μ) be topological spaces, let $x_0 \in X$ and $y_0 \in Y$, and let h = $(X, x_0) \to (Y, y_0)$ be a map. Then prove that h induces a homeomorphism $h_* = \pi_1(X, x_0) \to \pi_1(Y, y_0)$.

14. a) Let (X, τ) be a topological space, let $x_0 \in X$ and let $[\alpha] \in \pi_1(X, x_0)$ then prove that there exists $[\overline{\alpha}] \in \pi_1(X, x_0)$ such that $[\alpha] \circ [\overline{\alpha}] = [\overline{\alpha}] \circ [\alpha] = [e]$, where e : $I \to X$ be the path defined by $e(x) = x_0, x \in I$.

    b) Let (X, τ) and (Y, μ) be topological spaces, let $x_0 \in X$ and $y_0 \in Y$ and let h, k : $(X, x_0) \to (Y, y_0)$ be maps such that h≃V vel x_0. Then prove that $h_* = k_*$.

15. a) Let (X, τ) be a topological space, let $x_0 \in X$ and let $e : I \to X$ be the path defined by $e(x) = x_0$ for each $x \in I$ then prove that $[\alpha] \circ [e] = [e] \circ [\alpha] = [\alpha], \forall [\alpha] \in \pi_1(X, x_0)$.

    b) Let (X, τ) be a path connected space and let $x_0, x_1 \in X$ then prove that $\pi_1(X, x_0)$ is isomorphic to $\pi_1(X, x_1)$.
