K16P 0425

Reg. No. : ....

Name : ....

Second Semester M.Sc. Degree (Regular/Supplementary/Improvement) Examination, March 2016
(2014 Admn. Onwards)
MATHEMATICS
MAT 2C10 : Partial Differential Equations and Integral Equations

Time : 3 Hours Max. Marks : 60

PART – A

Answer **four** questions from this part. **Each** question carries **3** marks.

1. Eliminate the arbitrary function F from the equation $z = xy + F(x^2 + y^2)$ and find the corresponding partial differential equation.
2. Show that the equations $f = xp - yq - x = 0$, $g = x^2p + q - xz = 0$ are compatible.
3. Classify the equation $u_{xx} + x^2u_{yy} = 0$, for all x.
4. State (with necessary assumptions) the heat conduction problem of an infinite rod.
5. If $y''(x) = F(x)$ and y satisfies the initial conditions $y(0) = y_0$ and $y'(0) = y'_0$, show that $y(x) = \int_0^x (x - \xi)\, F(\xi)\, d\xi + y'_0\, x + y_0$.
6. Show that the characteristic numbers of a Fredholm equation with a real symmetric Kernel are all real. **(4×3=12)**

PART – B

Answer **any four** questions from this part without omitting any Unit. **Each** question carries **12** marks.

UNIT – I

7. a) Prove that a necessary and sufficient condition that the Pfaffian differential equation $\vec{X}.d\vec{r} = P(x, y, z)\, dx + Q(x, y, z)dy + R(x, y, z)dz = 0$ be integrable is that $\vec{X}.\ \text{curl}\ \vec{X} = 0$.

   b) Verify that the equation $yzdx + 2xzdy - 3xydz = 0$ is integrable and find the corresponding integral.

P.T.O.

8. a) Use Charpit's method to find the complete integral of $p^2x + q^2y = z$.

   b) Solve $p = (z + qy)^2$ by Jacobi's method.

9. a) Find the complete integral of $p^2x + qy - z = 0$ and derive the equation of the integral surface containing the line $y = 1, x + z = 0$.

   b) Solve the Cauchy problem for $2z_x + yz_y = z$ for the initial data curve $C : x_0 = s, y_0 = s^2, z_0 = s, 1 \le s \le 2$.

UNIT – II

10. a) Reduce the equation $(n-1)^2 u_{xx} - y^{2n} u_{yy} = n\, y^{2n-1} u_y$, where n is a positive integer to a canonical form and hence solve it.

    b) Derive d' Alembert's solution which describes the vibrations of an infinite string.

11. a) Prove that for the equation $Lu = u_{xy} + \frac{1}{4}u = 0$, the Riemann function is $v(x, y; \alpha, \beta) = J_0\left(\sqrt{(x-\alpha)(y-\beta)}\right)$ where J_0 is the Bessel's function of the first kind of order zero.

    b) Prove that the solution of the following problem if it exists is unique

    $u_{tt} - c^2u_{xx} = F(x, t), 0 < x < l, t > 0$

    $u(x, 0) = f(x), 0 \le x \le l$

    $u_t(x, 0) = g(x), 0 \le x \le l$

    $u(0, t) = u(l, t) = 0, t \ge 0.$

12. a) State Dirichlet problem for the upper half plane and solve it.

    b) Solve :

    $u_t = k\, u_{xx}, 0 < x < l, t > 0$

    $u(0, t) = u(l, t) = 0, t > 0$

    $u(x, 0) = f(x), 0 \le x \le l$

UNIT – III

13. Transform the boundary value problem $\frac{d^2y}{dx^2} + \lambda y = 0$, $y(0) = 0$, $y(l) = 0$ to a Fredholm equation of the second kind. Also recover the boundary value problem from the integral equation you obtain.

14. a) Show that the characteristic functions corresponding to distinct characteristic numbers of the equation $y(x) = \lambda \int_0^1 K(x, \xi)\, y(\xi)\, d\xi$ are orthogonal over the interval (0, 1) where $K(x, \xi)$ is symmetric.

    b) Show that the characteristic values of λ for the equation $y(x) = \lambda \int_0^{2\pi} \sin(x+\xi)\, y(\xi)\, d\xi$ are $\lambda_1 = 1/\pi$ and $\lambda_2 = -1/\pi$ with corresponding characteristic functions of the form $y_1(x) = \sin x + \cos x$ and $y_2(x) = \sin x - \cos x$.

15. a) Describe the method of successive approximations of solving a Fredholm equation of the second kind, $y(x) = F(x) + \lambda \int_a^b K(x, \xi)\, y(\xi)\, d\xi$.

    b) Apply the method of successive approximations to solve the equation $y(x) = 1 + \int_0^1 (1 - 3x\xi)\, y(\xi)\, d\xi$. **(4x12=48)**
