K16P 0467

Reg. No. : ....

Name : ....

Second Semester M.Sc. Degree (Regular/Supplementary/Improvement) Examination, March 2016

(2013 and Earlier Admn.)

MATHEMATICS

Paper – VII : Real Analysis – II

Time : 3 Hours Max. Marks : 60

Instructions : *1) Notations are as in prescribed **textbook**.*

*2) Answer **any four** questions from Part – **A**. **Each** question carries 3 marks.*

*3) Answer **any four** questions from Part – **B** without omitting **any** Unit. **Each** question carries 12 marks.*

PART – A

1. Show that the Lebesgue outer measure of a countable subset of $\mathbb{R}$ is zero.
2. Let f be a nonnegative measurable function with $\int f = 0$. Show that $f = 0$ a.e.
3. Let $f(0) = 0$ and $f(x) = x^2 \sin(1/x^2)$ for $x \neq 0$. Is it a function of bounded variation in $[-1, 1]$? Justify.
4. Let C [0, 1] be the space of all continuous functions on [0, 1]. For $f \in C[0, 1]$, define $\|f\| = \max |f(x)|$. Show that C [0, 1] is a Banach space under the norm $\|.\|$.
5. Give an example to show that the Hahn decomposition need not be unique.
6. Let E be a set for which $\mu \times v(E) = 0$. Then for almost all x, show that $v(E_x) = 0$.

(4×3=12)

PART – B

Unit – 1

7. a) Define Lebesgue outer measure. Show that the outer measure of an interval is its length.

   b) Deduce that [0, 1] is not countable.

8. a) Define a measurable function. Show that sum and product of two measurable functions are measurable.

   b) Give an example of a measurable and a non-measurable function.

P.T.O.

9. a) State and prove bounded convergence theorem.
   b) State and prove Fatou's lemma.
   c) Show that we may have strict inequality in Fatou's lemma.

Unit – 2

10. Let f be an increasing real valued function on the interval [a, b]. Show that f is differentiable a.e., that the derivative f′ is measurable and $\int_a^b f'(x)\,dx \le f(b) - f(a)$.

11. a) State and prove Hölder inequality. When does the equality hold ?
    b) Show that even if $0<p<1$, we have $f + g \in L^p$ for $f, g \in L^p$.

12. State and prove Riesz Representation Theorem.

Unit – 3

13. a) If $\mathcal{B}$ is a σ-algebra and μ is a measure, show that $\mu\left(\bigcap_{n=0}^{\infty} E_n\right) = \lim_{n\to\infty} \mu(E_n)$ for $E_i \in \mathcal{B}$, $\mu(E_1) < \infty$ and $E_i \supset E_{i+1}$.
    b) If $\{A_n\}$ is a collection of measurable sets, show that

$$\mu\left(\bigcup_{k=1}^{\infty} A_k\right) = \lim_{n\to\infty} \mu\left(\bigcup_{k=1}^{n} A_k\right),$$ where μ is a measure.

14. a) Define a positive set w.r.t. a signed measure. If E is measurable set such that $0 < vE < \infty$, show that there is a positive set A contained in E with $vA > 0$.
    b) State and prove Hahn decomposition theorem.

15. a) Let $(X, \mathcal{A}, \mu)$ and $(Y, \mathcal{B}, v)$ be two complete measure spaces and let $\mathcal{R}$ be the collection of all measurable rectangles. Assume that E is a set in $\mathcal{R}_{\sigma\delta}$ with $\mu \times v(E) < \infty$. Show that the function g defined by $g(x) = vE_x$ is a measurable function of x and $\int g\,d\mu = \mu \times v(E)$, where E_x is the x cross section.
    b) Establish the same conclusion when E is a measurable subset of X × Y.

(4×12=48)

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