M 27247

Reg. No. : ....

Name : ....

II Semester M.A./ M.Sc./ M.Com. Degree (Reg./Sup./Imp.) Examination, March 2015
MATHEMATICS
(2014 Admn. Onwards)
MAT 2C10 : Partial Differential Equations and Integral Equations

Time : 3 Hours Max. Marks : 60

PART – A

Answer **four** questions from this Part. **Each** question carries **3** marks.

1. Define a partial differential equation and explain the classification of them with suitable examples.

2. What do you mean by Cauchy Problem ?

   Solve $u_x + y\, u_y = u^2$ with the initial condition $u(0, y) = \sin y$.

3. Classify each of the following equation as hyperbolic, parabolic or elliptic.

   a) Wave propagation : $u_{tt} = c^2 u_{xx}$, $c > 0$

   b) Heat equation : $u_t = c\, u_{xx}$, $c > 0$

   c) Laplace's equation : $\Delta u = u_{xx} + u_{yy} = 0$

4. Solve the initial value problem :

   $u_{tt} = 9\, u_{xx}$

   $u(x, 0) = \cos x$

   $u_t(x, 0) = 0.$

5. State Neumann problem for the upper half plane. Also find its solution.

P.T.O.

6. Show that the homogeneous integral equation $y(x) = \lambda \int_0^1 (3x - 2)\xi\, y(\xi)d\xi$ has no characteristic numbers and eigen functions.

PART – B

Answer **any four** questions from this part without omitting any Unit. **Each** question carries 12 marks.

UNIT – I

7. a) Consider the equation $u_x + y\, u_y = 0$. Is there a solution satisfying the extra condition ?

 a) $u(x, 0) = 1$

 b) $u(x, 0) = x$?

 If yes, give a formula, if no, explain why ?

 b) Find a complete integral of $f = (p^2 + q^2)y - qz = 0$.

8. a) Find the integral surface of the equation

 $(2xy - 1)\, p + (z - 2x^2)q = 2\,(x - yz)$

 which passes through the line $x_0(s) = 1$, $y_0(s) = 0$, $z_0(s) = s$.

 b) Explain the method for solving the semi-linear equation.

 $P(x, y)\, p + Q(x, y)\, q = R(x, y, z)$ by the method of characteristics.

9. a) Prove that the Pfaffian differential equation

 $P(x, y, z)\, dx + Q(x, y, z)\, dy + R(x, y, z)\, dz = 0$ is integrable iff $\vec{X}.\text{curl}\, \vec{X} = 0$,

 where $\vec{X} = (P, Q, R)$.

 b) What is Monge cone ? What is the analytic expression of Monge cone ?

UNIT – II

10. a) Derive the d' Alembert's solution for the Vibrations of an infinite string.

 b) Find the general solution by the method of separation of variables : $u_{xy} + u = 0$.

11. a) Solve the following heat equation with boundary conditions :

 $u_t = k\, u_{xx}$

 $$u(0, t) = u(L, t) = 0,\ u(x, 0) = \begin{cases} 1 & 0 \le x < L/2 \\ 2 & L/2 \le x \le L \end{cases}$$

 b) Prove that the solution of the Neumann problem is unique up to the addition of a constant.

12. a) State and prove maximum and minimum principles for harmonic functions.

 b) State Dirichlet problem for a rectangle and derive its solution.

UNIT – III

13. a) Solve the homogeneous Fredholm integral equation of the second kind.

 $$y(x) = \lambda \int_0^{2\pi} \sin(x + \xi)\, y(\xi)\, d\xi$$

 b) Find the iterated Kernel for the Kernel

 $k(x, \xi) = \sin(x - 2\xi),\ 0 \le x \le 2\pi,\ 0 \le t \le 2\pi$

14. a) Using the method of successive approximations, solve the integral equation.

 $y(x) = 1 + \int_0^x y(\xi)d\xi$, taking $y_0(x) = 0$.

 b) Show that the characteristic numbers of a Fredholm integral equation with real symmetric kernel are real.

15. a) Construct the Green's function for the boundary value problem $y'' - y = 0$, $y(0) = y'(0)$ and $y(l) + \lambda y'(l) = 0$.

 b) Convert the following initial value problem into Voltera integral equation

 $y'' + y = 0,\ y(0) = 0,\ y'(0) = 1.$

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