M 27246

Reg. No. : ....

Name : ....

II Semester M.A./M.Sc./M.Com. Degree (Reg./Sup./Imp.) Examination, March 2015
(2014 Admn. Onwards)
MATHEMATICS
MAT 2C 09 : Complex Analysis

Time : 3 Hours   Max. Marks : 60

PART – A

Answer **four** questions from this Part. **Each** question carries **3** marks :

1. Show that $u(z) = \log |z|$ is harmonic on $G = \mathbb{C} - \{0\}$. Does there exist a harmonic conjugate of $u(z)$ on G ? Justify your answer.

2. Prove that $\int_0^{2\pi} \frac{e^{is}}{e^{is} - z} ds = 2\pi$ if $|z| < 1$.

3. Let $f(z) = \frac{1}{(z-1)(z-2)}$, give the Laurent expansion of f(z) valid in each of the annuli

   i) (0; 1, 2)   ii) (0; 2, ∞)

4. Show that for $a > 1$ $\int_0^{\pi} \frac{d\theta}{a + \cos\theta} = \frac{\pi}{\sqrt{a^2 - 1}}$.

5. Show that $\prod_{n=2}^{\infty} \left(1 - \frac{1}{n^2}\right) = \frac{1}{2}$.

6. Show that $\frac{d}{dz}\left(\frac{\Gamma'(z)}{\Gamma(z)}\right) = \sum_{n=0}^{\infty} \frac{1}{(n+z)^2}$.

P.T.O.

PART – B

Unit – I

Answer **any four** questions from this Part without omitting **any** unit.
Each question carries **12** marks :

7. a) If G is open and connected and $f = G \to \mathbb{C}$ is differentiable with $f'(z) = 0$ for all z in G, then prove that f is constant.

   b) Let G be either the whole plane or some open disk. If $u : G \to \mathbb{R}$ is a harmonic function, then prove that u has a harmonic conjugate.

8. a) Let f be analytic in B (a; R). Prove that $f(z) = \sum_{n=0}^{\infty} a_n (z-a)^n$ for $|z - a| < R$ where $R_n = \frac{1}{n!} f^{(n)}(a)$ and this series has radius of convergence $\geq R$.

   b) Let f be analytic in the open disk B(a; R) and suppose that r is a closed rectifiable curve in B(a; R). Prove that $\int_r f = 0$.

9. a) If $r : [0, 1] \to \mathbb{C}$ is a closed rectifiable curve and r does not pass through a, then prove that $\frac{1}{2\pi i} \int_r \frac{dz}{z-a}$ is an integer.

   b) Let G be an open set and let $f : G \to \mathbb{C}$ be a differentiable function, prove that f is analytic on G.

Unit – II

10. a) If f has an essential singularity at z = a, then prove that f(z) comes arbitrarily close to every complex number as z approaches a .

    b) Show that $\int_0^{\infty} \frac{\sin x}{x} dx = 0$.

11. a) State and prove Argument principle.

    b) State and prove Rouche's theorem and deduce the fundamental theorem of algebra.

12. a) State the second version of the maximum modulus theorem. What happens if the boundedness condition is dropped ? Justify.

    b) State Schwarz's Lemma. If $f: D \to D$ is a one to one analytic map of D onto itself and if f(a) = 0, prove that there is a complex number c with |e| = 1 such that $f = C\varphi_a$.

Unit – III

13. a) With the usual notations prove that $C(G, \Omega)$ is a complete metric space.

    b) Define the set H(G). If $\{f_n\}$ is a sequence in H(G) and f belongs to $C(G, \mathbb{C})$ such that $f_n \to f$, then prove that $f_n^{(k)} \to f^{(k)}$ for each integer $k \geq 1$.

14. State and prove Riemann mapping theorem.

15. a) State and prove Weierstrass factorization theorem.

    b) Obtain the factorization of the sine function.

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