M 27245

Reg. No. : ....

Name : ....

II Semester M.A./M.Sc./M.Com. Degree (Reg./Sup./Imp.)
Examination, March 2015
(2014 Admn. Onwards)
MATHEMATICS
MAT2C08 : Topology

Time : 3 Hours Max. Marks : 60

Instructions: *Answer* ***four*** *questions from Part* ***A***. ***Each*** *question carries* ***3*** *marks.*

Answer ***four*** *questions from Part* ***B*** *without omitting any Units.* ***Each*** *question carries* ***12*** *marks.*

PART – A

1. Give an example of a topological space, which is regular but not completely regular.

2. Let $X = \{1, 2, 3, 4\}$ determine whether each of the following topological spaces is normal.

   i) (X, τ), where $\tau = \{\phi, \{1, 2\}, \{3, 4\}, X\}$

   ii) (X, τ), where $\tau = \{\phi, \{1\}, \{1, 2\}, \{1, 2, 3\}, X\}$.

3. Let X be a set, let $\{A_\alpha : \alpha \in \wedge\}$ be a collection of subsets of X, let τ be the weak topology as X induced by A, and let (Y, U) be a topological space. Prove that a function $f : X \to Y$ is continuous if and only if for each $\alpha \in \wedge$, $f/_{\wedge_\alpha} = A_\alpha \to Y$ is continuous.

4. Let (X, τ) be a topological space and let (Y, u) be its one-point compactification. Prove that X is a dense subset of Y if and only if X is not compact.

P.T.O.

5. When will a topological space X be contractible ? Illustrate it with an example.

6. Define deformation retract of a topological space X and prove that the unit circle S′ of $\mathbb{R} \times \mathbb{R}$ is the deformation retract of the closed annulus $X = \{(x, y) \in \mathbb{R} \times \mathbb{R} : 1 \le x^2 + y^2 \le 4\}$.

PART – B

UNIT – I

7. i) Let $A = \{\eta_n : n \in \mathbb{N}\}$ and let $\mathcal{B} = \{B \in \wp(\mathbb{R}) = B$ is an open interval that does not contain 0 or there is a positive number x such that $B = (-x, x) - 4\}$. Then prove that (a) $\mathcal{B}$ is a basis for a topology τ on $\mathbb{R}$ the space $(\mathbb{R}, \tau)$ is a Hausdorff space.

   ii) Characterise regular spaces.

8. i) Define complete regularity and prove that every metric space is completely regular.

   ii) Define Moore plane and prove that the Moore plane is not normal.

9. i) Let (X, τ) be a topological space and let (Y, u) be a Hausdorff space and let f and $g : X \to Y$ be continuous functions. If f and g agree on a dense subset of X then prove that f = g.

   ii) Prove that Hausdorff space (X, τ) is locally compact of and only if for each $p \in X$ and each neighborhood V of p there is a neighborhood U of p such that $\overline{U}$ is compact and $\overline{U} \subset V$.

UNIT – II

10. i) Let $(\Omega, \le)$ be an uncountable well ordered set with a maximal element w_1 having the property that if $x \in \Omega$ and $x \ne w_1$ then $\{y \in \Omega : y \le x\}$ is countable let τ be the order topology of Ω and let $\Omega_0 = \Omega = [w_1]$. Prove that $(\Omega_0, \tau_{\Omega_0})$ is countably compact but not compact.

    ii) Define local compactness in topological spaces. If (X, τ) is a locally compact Hausdorff space then prove that for each point $p \in X$ and for each neighborhood V of p there is an open set U ad a compact subset k of X such that $p \in u, u \le v, u \le k$ and $k \subset \overline{U}$.

11. i) When is a topological space X said to be a k-space. Illustrate with an example.

    ii) Prove that the topological product of countable collective of metric spaces is metrizable.

12. Let (X, τ) be a topological space and let s be a sub basis for τ. If every cover of X by members of τ has a finite subcover then X is compact.

UNIT – III

13. i) Let (X, τ), (Y, U) and (Z, v) be topological spaces, let $f_1, f_2 : X \to Y$ be continuous functions such that $f_1 \simeq f_2$ and let $g_1, g_2 : Y \to Z$ be continuous functions such that $g_1 \simeq g_2$ that $g_1 \circ f_1 \simeq g_2 \circ f_2$.

    ii) Determine the fundamental group of a convex subset of the Euclidean space $\mathbb{R}^n$.

    iii) Let (X, τ) and (Y, u) be topological spaces, let $h : X \to Y$ be a homeomorphism, let $x_0 \in X$ and let $y_0 = h(x_0)$. Then prove that $\pi_1(X_1\ x_0)$ is isomorphic to $\pi_1(Y_1\ y_0)$.

14. i) Let (X, τ) be a topological space and let $x_0 \in X$. Furthermore, let $\alpha_1, \alpha_2, \beta_1, \beta_2 \in \Omega(X, x_0)$ and suppose $\alpha_1 \simeq_p \alpha_2$ and $\beta_1 \simeq_p \beta_2$. Then prove that $\alpha_1 * \beta_1 \simeq_p \alpha_2 * \beta_2$.

    ii) Let (X, τ) and (Y, u) be topological spaces, let $x_0 \in X$ and $y_0 \in Y$ and let $h : (X, x_0) \to (Y, y_0)$ be a map. Establish that the homeomorphism h induces a homeomorphism has $\pi_1(X, x_0) \to \pi_1(Y, y_0)$.

15. i) State and prove covering path property.

    ii) Is S′ a contractible space ? Why ?
