M 27341

Reg. No. : ....

Name : ....

Second Semester M.A./M.Sc./M.Com. Degree (Regular/Supplementary/ Improvement) Examination, March 2015
(2013 & Earlier Admn.)
MATHEMATICS
Paper – X : Partial Differential Equations

Time : 3 Hours Max. Marks : 60

PART – A

Answer **any four** questions. **Each** question carries **3** marks :

1. a) Find the general integral of $(x^3 + 3xy^2)p + (y^3 + 3x^2y)q = 2(x^2 + y^2)z$.
   b) Show that the equation $z = px + qy$ is compatible with any equation $f(x, y, z, p, q) = 0$.
   c) Find a complete integral of $p^2 + q^2 = x + y$.
   d) Reduce the equation $4u_{xx} - 4u_{xy} + 5u_{yy} = 0$ into Canonical form.
   e) State the interior dirichelet problem. Prove that the solution of the interior dirichelet problem is unique.
   f) State interior Neumann problem. Establish a necessary condition for the existence of the solution of the interior Neumann problem.

PART – B

Answer **any four** questions without omitting any unit. **Each** question carries **12** marks.

UNIT – I

2. a) Eliminate the parameters a and b from the equation $z^2(1 + a^3) = 8(x + ay + b)^3$ and find the corresponding p.d.e.
   b) Prove that the general solution of the quasi-linear equation $P(x, y, z)p + Q(x, y, z)q = R(x, y, z)$ where P, Q and R are continuously differentiable functions of x, y and z is $F(u, v) = 0$ where F is an arbitrary differentiable function of u and v and $u(x, y, z) = c_1$ and $v(x, y, z) = c_2$ are two independent solutions of the system $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$.

3. a) Prove that a Pfaffian differential equation is two variables always possesses an integrating factor.
   b) Prove that the equation $yzdx + xzdy + xydz = 0$ is integrable. Find its primitive.

P.T.O.

4. a) Find the surface which intersect the surface of the system $z(x + y) = c(3z + 1)$ orthogonally and which passes through the circle $x^2 + y^2 = 1$, $z = 1$.

   b) Eliminate the arbitrary function F from the equation $F(x - z, y - z) = 0$.

UNIT – II

5. a) Explain Charpit's method to find a complete integral of a first order PDE.

   b) Find a complete integral of $z^2 + zu_z - u_x^2 - u_y^2 = 0$ by Jacobi's method.

6. a) Solve the equation $r = t$, by Monge's method.

   b) Solve the equation $\left(D^2 - D'^2 - 3D + 3D'\right)u = e^{x+2y}$.

7. a) Find by the method of characteristics, the integral surface of $pq = xy$ which passes through the curve $z = x$, $y = 0$.

   b) Solve the equation $rq^2 - 2pqs + tp^2 = pt - qs$.

UNIT – III

8. a) Discuss the Dirichelet's problem for a sphere.

   b) State Kelvin's inversion theorem.

9. a) Solve the one dimensional diffusion equation $/\theta_{xx} = \frac{1}{k}\theta_t$ in the range $0 \le x \le 2\pi$ subject to the conditions :

   $\theta(x, 0) = \sin^2 x, \; 0 \le x \le 2\pi$

   $\theta(0, t) = \theta(2\pi, t) = 0, \; t \ge 0$

   b) A uniform insulated sphere of dielectric constant k and radius a carries on its surface a charge of density $\lambda P_n(\cos\theta)$. Prove that the interior of the sphere contributes an amount $8\pi^2 x^2 a^3 kn/(2n+1)(kn+n+1)^2$ to the electrostatic energy.

10. a) Find the potential function $\varphi(x, y, z)$ in the region $0 \le x \le a, 0 \le y \le b, 0 \le z \le c$ satisfying the conditions :

    i) $\varphi = 0$ on $x = 0, x = a, y = 0, y = b, z = 0$

    ii) $\varphi = f(x, y)$ on $z = c, \; 0 \le x \le a, \; 0 \le y \le b$

    b) Obtain the solution of the boundary value problem :

    $\theta_{xx} = \frac{1}{k}\theta_t, \; 0 \le x < \infty$

    $\theta(x, 0) = f(x), \; x > 0.$

    $\theta(0, t) = 0, \; t > 0.$

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