M 27339

Reg. No. : ....

Name : ....

II Semester M.A./M.Sc./M.Com. Degree (Regular/Supplementary/Improvement) Examination, March 2015

MATHEMATICS

(2013 and Earlier Admn.)

Paper VIII : Topology – II

Time : 3 Hours   Max. Weightage : 60

Instructions : *1) Answer **any four** questions from Part **A** and **Each** question carries **3** marks.*

*2) Answer **any four** questions from Part **B** without omitting **any** Unit and **each** question carries **12** marks.*

PART – A   **(4×3=12)**

1. Define a directed set. Give an example.

2. Prove that continuous function preserves local compactness.

3. Show that $| d(x, z) - d(y, z) | \leq d(x, y)$ for x, y, z in a metric space M.

4. Define co-finite filter and atomic filter.

5. Give an example of a second countable space.

6. Define Net and describe its convergence.

PART – B   **(4×12=48)**

UNIT – I

7. a) Let $\{f_i : X \to Y_i \mid i \in I\}$ be a family of functions which distinguishes points from closed sets in X. Show that the evaluation map $e : X \to \Pi_{i \in I} Y_i$ is open when regarded as a function from X onto e(X).

   b) Prove that metrizability is a countably productive property.

P.T.O.

8. a) Let $X = \Pi_{i \in I} X_i$, where X_i being a topological space. Show that the sequence $\{x_n\}$ converges to x in X, if and only if for each $i \in I$, the sequence $(\pi_i(x_n))$ converges to $\pi i(x)$ in X_i.

   b) Prove that the topological product is regular if and only if each co-ordinate space is regular.

9. a) Show that a topological space is Tychonoff space if and only if it is embeddable into a cube..

   b) Show that a product space is locally connected if and only if each co-ordinate space is locally connected.

UNIT – II

10. a) Prove that every filter is contained in an ultrafilter.

    b) Prove that a first countable, countably compact space is sequentially compact.

11. a) Prove that a subset B of a space X is open if and only if no net in B^C can converge to a point in B.

    b) Prove that the product space is compact if and only if each factor space is compact.

12. a) Prove that every countably compact metric space is second countable.

    b) Prove that a space is Hausdorff if and only if every ultrafilter converges to at most one point.

UNIT – III

13. a) Prove that a metric space is compact if and only if it is complete and totally bounded.

    b) Prove that every metric space can be isometrically embedded as a dense subspace of a complete metric space.

14. a) Define compactification of a topological space. Show that among all Hausdorff compactifications of a Tychonoff space, the Stone-Cech compactification if the largest one upto topological equivalence.

    b) Prove that every locally compact, Hausdorff space is regular.

15. a) Prove that any compact subset of a Hausdorff space is closed.

    b) Prove that a subset of first category in a complete metric space cannot have any interior points.
