M 27337

Reg. No. : ....

Name : ....

Second Semester M.A./M.Sc./M.Com. Degree (Regular/Supplementary/ Improvement) Examination, March 2015
MATHEMATICS
(2013 and Earlier Admn.)
Paper – VI : Algebra – II

Time : 3 Hours Max.Marks : 60

Instructions : *1) Answer* ***any 4*** *questions from Part* ***A****.*
2) ***Each*** *question carries* ***3*** *marks.*
3) Answer any 4 questions from Part – B without omitting any Unit. ***Each*** *question carries* ***12*** *marks.*

PART – A

Answer **any 4** questions. **Each** question carries **3** marks.

1. Prove that if p is an irreducible in a UFD, then p is a prime.
2. Prove that 6 does not factor uniquely (upto units) into irreducibles in $Z[\sqrt{-5}]$. Give two different factorizations.
3. Find an extension field of $\mathbb{Z}_2$, containing a zero α of $x^2 + x + 1$ in $\mathbb{Z}_2[x]$.
4. Find the fixed field of $\psi_{\sqrt{2}, -\sqrt{2}}$ of $\mathbb{Q}(\sqrt{2})$.
5. Find the splitting field of $x^4 - 5x^2 + 6$ over $\mathbb{Q}$.
6. If E is a finite extension of a field E, then prove that {E : F} divides [E : F]. **(4×3=12)**

PART – B

Answer **any 4** questions without omitting **any** Unit. **Each** question carries **12** marks.

Unit – I

7. a) Define : Principal Ideal Domain. Give one example.
   b) Prove that every PID is a UFD.
   c) Prove that the integral domain $\mathbb{Z}$ is a UFD.

P.T.O.

8. a) Prove that for a Euclidean domain with a Euclidean valuation ν, $\nu(1)$ is minimals among all $\nu(a)$ for non-zero $a \in D$, and $u \in D$ is a unit if and only if $\nu(u) = \nu(1)$.

   b) State the Euclidean algorithm to find the gcd of two integers a and b. Find the gcd of 22,471 and 3,266 and express it in the form $\lambda(22471) + 3(3266)$ for $\lambda, \mu, \in Z$.

9. a) Prove that Z [i] is a Euclidean domain.

   b) Let p be an odd prime in $\mathbb{Z}$, and $p = a^2 + b^2$ for integers a and b in $\mathbb{Z}$. Prove that $p \equiv 1 \pmod 4$

Unit – II

10. a) Prove that a finite extension field E of a field F is an algebraic extension of F.

    b) If E is a finite extension field of a field F, and K is a finite extension field of E, then prove that K is a finite extension of F, and [K : F] = [K : E] [E : F].

11. a) Prove that a field F is algebraically closed if and only if every non-constant polynomial in F [x] factors in F [x] into linear factors.

    b) Prove that the field $\mathbb{C}$ of complex numbers is an algebraically closed field.

    c) Find a basis of $\mathbb{Q}(\sqrt{2}, \sqrt{3})$ over $\mathbb{Q}$.

12. a) Let E be a finite extension of degree n over a finite field F. Prove that if F has q elements, then E has q^n elements.

    b) Prove that a finite extension E of a finite field F is a simple extension of F.

    c) If F is a finite field, then prove that for every positive integer n, there is an irreducible polynomial in F [x] of degree n.

Unit – III

13. a) Define : The splitting field of f(x) over a field F. Give one example.

    b) Prove that a field E, where $F \leq E \leq \overline{F}$, is a splitting field over F, if and only if every automorphism of $\overline{F}$ leaving F fixed maps E onto itself and this induces an automorphism of E leaving F fixed.

14. a) Let $\overline{F}$ be an algebraic closure of F, and let $f(x) = x^n + a_{n-1}x^{n-1} + ... + a_1 x + a_0$ be any monic polynomial in $\overline{F}[x]$. Prove that if $(f(x))^m \in F[x]$ and $m.1 \neq 0$ in F, then $f(x) \in F[x]$.

    b) Prove that every field of characteristic '0' is perfect.

15. a) State the main theorem of Galois theory.

    b) Show by an example that the lattice of subgroups of G(K/F) is the invorted lattice of intermediate fields of K over F.

    c) Let K be a finite extension of degree n of a finite field F of P^r elements. Prove that G (K/F) is cyclic of order n, and is generated by σ_{p^r} where $\sigma_{p^r}(\alpha) = \alpha^{p^r}$ for $\alpha \in K$. **(4×12=48)**

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