Reg. No. : ....

Name : ....

M 25134

II Semester M.A./M.Sc./M.Com. Degree (Reg./Sup./Imp.) Examination, March 2014
MATHEMATICS
Paper – VII : Real Analysis – II

Time : 3 Hours Max. Marks : 60

Instructions: *1) There are Part **A** and Part **B**. Part **A** has **6** questions out of which **4** are to be answered. Part **B** has **9** questions out of which **4** are to be answered without omitting **any** Unit.*

*2) **Each** question of Part **A** carries **3** marks and Part **B** carries **12** marks.*

PART – A

Answer **any four** questions. **Each** question carries **3** marks.

1. Prove that the outer measure m^* is translation invariant.

2. Show that the sum and product of two simple functions are simple.

3. Show that we may have strict inequality in Fatou's Lemma.

4. Show that the sum of two absolutely continuous functions is absolutely continuous.

5. Let C = C [0, 1] be the space of all continuous functions on [0, 1] and define $\|f\| = \max |f(x)|$. Show that $\subset$ is a Banach space.

6. Define complete measure. Give an example. **(4×3=12)**

PART – B

Answer **any four** questions without omitting **any** Unit. **Each** question carries **12** marks.

UNIT – 1

7. a) Show that the family m of measurable sets is an algebra of sets.

   b) Show that the interval $(0, \infty)$ is measurable.

P.T.O.

8. a) Construct a non measurable set.

b) If f is a measurable function and f = g a.e., then prove that g is measurable.

9. a) State and prove Fatou's Lemma.

b) Let g be integrable over E and let $\langle f_n \rangle$ be a sequence of measurable functions such that $|f_n| \leq g$ on E and for almost all $x \in E$ we have $f(x) = \lim f_n(x)$. Then prove that $\int_E f = \lim \int_E f_n$.

UNIT – 2

10. a) Let f be an increasing real-valued function on the interval [a, b]. Then prove that f is differentiable almost every where and $\int_a^b f'(x)\,dx \leq f(b) - f(a)$.

b) Define functions of bounded variation. Give an example.

11. a) Show that a normed linear space X is complete if and only if every absolutely summable series is summable.

b) If f and g are in L^P with $1 \leq p \leq \infty$, then prove that $f + g \in L^P$.

12. a) Given $f \in L^p$, $1 < p \leq \infty$ and $\varepsilon > 0$, then prove that there is a step function ϕ and a continuous function ψ such that $\| f - \phi \|_p < \varepsilon$ and $\| f - \psi \|_p < \varepsilon$.

b) Let F be a bounded linear functional on L^p, $1 \leq p < \infty$. Then prove that there is a function g in L^ε such that $F(f) = \int fg$.

UNIT – 3

13. a) If $E_i \in \mathcal{B}$, $\mu(E_1) < \infty$ and $E_i > E_{i+1}$, then prove that $\mu\left(\bigcap_{i=1}^{\infty} E_i\right) = \lim_{n \to \infty} \mu(E_n)$.

b) Let $(x, \mathcal{B})$ be a measurable space, $\langle \mu_n \rangle$ a sequence of measures that converge setwise to a measure μ, and $\langle f_n \rangle$ a sequence of nonnegative measurable functions that converge pointwise to the function f. Then prove that $\int f d_\mu \leq \underline{\lim} \int f_n d\mu_n$.

14. a) Prove that the union of a countable collection of positive sets is positive.

b) State and prove Lebesgue decomposition theorem.

15. a) Define measure and outer measure. Let μ be a measure on an algebra G, μ^* the outer measure induced by μ and E any set. Then for $\varepsilon > 0$, prove that there is a set $A \in G_\sigma$ with $E \subset A$ and $\mu^* A \leq \mu^* E + \varepsilon$. Also prove that there is a set $B \in G_{\sigma\delta}$ with $E \subset B$ and $\mu^* E = \mu^* B$.

b) State and prove Tonelli's theorem. **(4×12=48)**

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