M 25135

Reg. No. : ....

Name : ....

II Semester M.A./M.Sc./M.Com. Degree (Reg./Sup./Imp.) Examination, March 2014
MATHEMATICS
Paper – VIII : Topology – II

Time: 3 Hours Max. Marks : 60

Instructions : *1) Answer* ***any four*** *questions from Part* ***A*** *and* ***each*** *question carries 3 marks.*

2) Answer ***any four*** *questions from Part* ***B*** *without omitting* ***any*** *Unit and* ***each*** *question carries* ***12*** *marks.*

PART – A (4×3=12)

1. Give an example of a T_1- space which is not a T_2-space.
2. Give an example of a first countable T_2-space which is not metrizable.
3. Describe the notion of Filter convergence.
4. Give an example of a second countable space.
5. Define Net and describe its convergence.
6. Define Riemann net.

PART – B (4×12=48)

Unit – I

7. a) Let $X = \Pi_{i\in I}X_i$, where X_i being a topological space. Show that the sequence $\{x_n\}$ converges to x in X, if and only if for each $i \in I$, the sequence $\{\pi_i(x_n)\}$ converges to $\pi_i(x)$ in X_i.

   b) Prove that the topological product is regular if and only if each co-ordinate space is regular.

P.T.O.

8. a) Prove that if the product of an indexed family of sets is non-empty, then each projection is onto.

   b) Prove that each co-ordinate space can be embeddable in a non-empty product space.

9. a) Prove that a topological space is second countable if and only if all co-ordinate spaces are so and all except finitely many are indiscrete spaces.

   b) Show that product space is locally connected if and only if each co-ordinate space is locally connected and all except finitely many of them are connected.

Unit – II

10. a) State and prove Alexander Sub-base theorem.

    b) Prove that a topological space is Hausdorff if and only if no filter can converge to more than one point in it.

11. a) Prove that sequential compactness is a countably productive property.

    b) Prove that a first countable, countably compact space is sequentially compact.

12. a) Prove that a subset of B of a space X is open if and only if no net in B^c can converge to a point in B.

    b) Prove that every filter is contained in an ultrafilter.

Unit – III

13. a) Define compactification of a topological space. Show that among all Hausdorff compactifications of a Tychonoff space, the Stone-Cech compactification is the largest one upto topological equivalence.

    b) Prove that every locally compact, Hausdorff space is regular.

14. a) Prove that a metric space is compact if and only if it is complete and totally bounded.

    b) Prove that a subset of first category in a complete metric space cannot have any interior points.

15. a) Prove that any compact subset of a Hausdorff space is closed.

    b) Prove that every metric space can be isometrically embedded as a dense subspace of a complete metric space.