0043091 K19P 1516

Reg. No. : ....

Name : ....

I Semester M.Sc. Degree (CBSS-Reg./Suppl./Imp.)
Examination, October - 2019
(2017 Admission Onwards)
MATHEMATICS
MAT1C01:BASIC ABSTRACT ALGEBRA

Time : 3 Hours Max. Marks : 80

PART-A

Answer any **Four** questions from this part. Each question carries **4** marks.

1. Prove that every ideal of $\mathbb{Z}$ is principal.
2. Give addition and multiplication tables of the ring $2\mathbb{Z}/8\mathbb{Z}$
3. Find all abelian groups up to of order 16.
4. Find the ascending central series of D_4.
5. Let X be a G-set. Then prove that $Gx = \{g \in G \mid gx = x\}$ is a subgroup of G.
6. Find the Sylow 3-subgroups of $\mathbb{Z}_{12}$.

PART - B

Answer **Four** questions from this part without omitting any unit. Each question carries **16** marks.

UNIT- I

7. a) Define decomposable group. Prove that the finite indecomposable abelian groups are exactly the cyclic groups with order a power of a prime.

   b) State and prove Cauchy's Theorem.

8. a) Let X be a G-set. For each $g \in G$, prove that the function $\sigma_g : X \to X$ defined by $\sigma_g(x) = gx$ is a permutation of X. Hence prove that the map $\phi : G \to S_X$ defined by $\phi(g) = \sigma_g$ is a homomorphism.

   b) For a prime p, prove that every group G of order p^2 is abelian.

P.T.O.

9. a) Let G be a group containing normal subgroups H and K such that $H \cap K = \{e\}$ and $H \vee K = G$. Then prove that G is isomorphic to H × K.

   b) Prove that no group of order 48 is simple.

UNIT-II

10. a) Let F be the field of quotient of an integral domain D. Then prove that the map $i: D \to F$ defined by i(a)=[(a, 1)] is an isomorphism of D with a subring of F.

    b) State and prove second isomorphism theorem in group theory.

11. a) Let K and L be normal subgroup of G with $K \vee L = G$, and $K \cap L = \{e\}$. Show that $G/K \simeq L \, and \, G/L \approx K$.

    b) If G has a composition series and N is a proper normal subgroup of G, then prove that there exists a composition series containing N. Give an example of a composition series.

12. a) Let G be a nonzero free abelian group with a finite basis. Then prove that every basis of G is finite and all basis have same number of elements.

    b) Show that a free abelian group contains no nonzero elements of finite order.

UNIT-III

13. a) If G be a finite subgroup of the multiplicative group $\langle F^* . \rangle$ of a field F. Then prove that G is cyclic.

    b) State and prove Eisenstein Criterion.

14. a) The polynomial X^4+4 can be factored into linear factors in $\mathbb{Z}_5[x]$. Find this factorization.

    b) State and Prove the evaluation homomorphism theorem for field theory.

15. a) Let R be a commutative ring with unity. Then prove that if M is a maximal ideal of R if and only if R/M is a field.

    b) If F is a field, prove that every non trivial prime ideal in F[x] is principal.
