Reg. No.:....

Name :

I Semester M.Sc. Degree (CBSS-Reg/Suppl./Imp.)
Examination, October - 2019
(2017 Admission Onwards)
MATHEMATICS

MAT1C01:BASIC ABSTRACT ALGEBRA

Time: 3 Hours Max. Marks: 80

PART-A

Answer any Four questions from this part. Each question carries 4 marks.

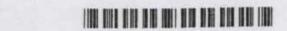
- 1. Prove that every ideal of \mathbb{Z} is principal.
- 2. Give addition and multiplication tables of the ring $2\mathbb{Z}/8\mathbb{Z}$
- 3. Find all abelian groups up to of order 16.
- Find the ascending central series of D₄.
- 5. Let X be a G-set. Then prove that $Gx = \{g \in G \mid gx = x\}$ is a subgroup of G.
- 6. Find the Sylow 3-subgroups of \mathbb{Z}_{12} .

PART - B

Answer Four questions from this part without omitting any unit. Each question carries 16 marks.

UNIT-I

- a) Define decomposable group. Prove that the finite indecomposable abelian groups are exactly the cyclic groups with order a power of a prime.
 - b) State and prove Cauchy's Theorem.
- 8. a) Let X be a G-set. For each g∈G, prove that the function σ_g: X → X defined by σ_g(x) = gx is a permutation of X. Hence prove that the map φ: G → S_x defined by φ(g) = σ_g is a homomorphism.
 - b) For a prime p, prove that every group G of order p2 is abelian.



9. a) Let G be a group containing normal subgroups H and K such that $H \cap K = \{e\}$ and $H \vee K = G$. Then prove that G is isomorphic to $H \times K$.

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b) Prove that no group of order 48 is simple.

UNIT-II

- 10. a) Let F be the field of quotient of an integral domain D. Then prove that the map i: D→F defined by i(a)=[(a, 1)] is an isomorphism of D with a subring of F.
 - b) State and prove second isomorphism theorem in group theory.
- 11. a) Let K and L be normal subgroup of G with $K \lor L = G$, and $K \cap L = \{e\}$. Show that $G / K \simeq L$ and $G / L \approx K$.
 - b) If G has a composition series and N is a proper normal subgroup of G, then prove that there exists a composition series containing N. Give an example of a composition series.
- a) Let G be a nonzero free abelian group with a finite basis. Then prove that every basis of G is finite and all basis have same number of elements.
 - Show that a free abelian group contains no nonzero elements of finite order.

UNIT-III

- 13. a) If G be a finite subgroup of the multiplicative group $\langle F^*.\rangle$ of a field F. Then prove that G is cyclic.
 - b) State and prove Eisenstein Criterion.
- 14. a) The polynomial X^4+4 can be factored into linear factors in $\mathbb{Z}_s[x]$. Find this factorization.
 - b) State and Prove the evaluation homomorphism theorem for field theory.
- 15. a) Let R be a commutative ring with unity. Then prove that if M is a maximal ideal of R if and only if R/M is a field.
 - b) If F is a field, prove that every non trivial prime ideal in F[x] is principal.