

0043594

K19P 1517



Reg. No. :

Name :

I Semester M.Sc. Degree (CBSS-Reg./Suppl./Imp.)

Examination, October - 2019

(2017 Admission Onwards)

MATHEMATICS

MAT 1C02 : LINEAR ALGEBRA



Time : 3 Hours

Max. Marks : 80

PART-A

Answer Four questions from this part. Each question carries 4 marks.

1. Let V be the real vector space of all functions f from \mathbb{R} into \mathbb{R} . Check whether the set of all f such that $f(-1)=0$ is a subspace or not.
2. Let W_1 and W_2 be subspaces of a vector space V such that the set-theoretic unions of W_1 and W_2 is also a subspace. Prove that one of the spaces W_i is contained in the other.
3. Let V be the space of $n \times 1$ matrices over F and let W be the space of $m \times 1$ matrices over F . Let A be a fixed $m \times n$ matrix over F and let T be the linear transformation from V into W defined by $T(X)=AX$. Prove that T is the zero transformation if and only if A is the zero matrix.
4. Let T be the linear operator on \mathbb{C}^2 defined by $T(x_1, x_2) = (x_1, 0)$. Let B be the standard ordered basis for \mathbb{C}^2 and let $B' = \{\alpha_1, \alpha_2\}$ be the ordered basis defined by $\alpha_1 = (1, i), \alpha_2 = (-i, 2)$, then what is the matrix of T in the ordered basis $\{\alpha_2, \alpha_1\}$?
5. Let V be a finite-dimensional vector space. What is the minimal polynomial for the identity operator on V ?
6. Let V be an inner product space. The distance between two vectors α and β in V is defined by $d(\alpha, \beta) = \|\alpha - \beta\|$. Then show that $d(\alpha, \beta) \leq d(\alpha, \gamma) + d(\gamma, \beta)$.

P.T.O.



PART-B

Answer 4 questions from this part without omitting any unit. Each question carries 16 marks.

UNIT-I

7. a) If W is a subspace of a finite-dimensional vector space V , then prove that every linearly independent subset of W is finite and is part of a basis for W .
- b) Let A and B be $m \times n$ matrix over the field F . Then prove that A and B are row equivalent if and only if they have the same row space.
- c) Define rank of a linear transformation.
8. a) Define invertible linear transformation and give an example. Let V and W be vector space over the field F and let T be a linear transformation from V into W . If T is invertible, then prove that the inverse function is a linear transformation from W into V .
- b) Let V , W , and Z be vector space over the field F . Let T be a linear transformation from V into W and U be a linear transformation from W into Z . If B, B' and B'' are ordered bases for the spaces V, W and Z , respectively, if A is the matrix of T relative to the pair B, B' and B is the matrix of U relative to the pair B, B'' , then prove that the matrix of the composition UT relative to the pair B, B'' is the product matrix $C=BA$.
9. a) Let V be a finite dimensional vector space over the field F , and let W be a subspace of V . Then show that $\dim W + \dim W^\circ = \dim V$.
- b) If S is any subset of a finite-dimensional vector space V , then show that $(S^\circ)^\circ$ is the subspace spanned by S .

UNIT-II

10. a) Define characteristic space.
- b) Let T be a linear operator on a finite dimensional space V . Suppose that $T\alpha = c\alpha$. If f is any polynomial, then prove that $f(T)\alpha = f(c)\alpha$.
- c) Let T be a linear operator on a n -dimensional space V , and suppose that T has distinct characteristic values. Prove that T is diagonalizable.



11. a) Define minimal polynomial for a linear operator on a finite dimensional vector space.
- b) Let T be a linear operator on a finite dimensional vector space V . If f is the characteristic polynomial for T , then prove that $f(T)=0$.
12. a) Let T is any linear operator on a vector space V and W be an invariant subspace for T . Then show that the characteristic polynomial for the restriction operator T_w divides the characteristic polynomial for T and the minimal polynomial for T_w divides the minimal polynomial for T .
- b) Let V be a finite dimensional vector space over the field F . Let F be a commuting family of triangulable linear operator on V . Then prove that there exists an ordered basis for V such that every operator in F is represented by a triangular matrix in that basis.

UNIT-III

13. a) Define projection operator.
- b) Let T be a linear operator on the space V , and let W_1, \dots, W_k and E_1, \dots, E_k satisfies.
- Each E_i is a projection.
 - $E_i E_j = 0$ if $i \neq j$;
 - $I = E_1 + \dots + E_k$;
 - The range of E_i is W_i .
- Then prove that a necessary and sufficient conditions that each subspace W_i be invariant under T is that T commutes with each of the projections E_i .
- c) If T is a linear operator on a finite dimensional vector space, then prove that every T -admissible subspace has a complementary subspace which is also invariant under T .
14. State and prove cyclic decomposition theorem.
15. a) State and prove Gram-Schmidt orthogonalization process.
- b) Define orthogonal projection and give an example.