



## Unit-III

13. a) Let  $(X, T)$  be a topological space and let  $A \subseteq X$ . Prove that the following conditions are equivalent.
- The subspace  $(A, T_A)$  is connected.
  - The set  $A$  cannot be expressed as the union of two non empty sets that are separated in  $X$ .
  - There do not exist  $U, V \in T$  such that  $U \cap A \neq \phi$ ,  $V \cap A = \phi$ ,  $U \cap V \cap A = \phi$  and  $A \subset U \cup V$ .
- b) Let  $T$  be the usual topology on  $\mathbb{R}$ . Prove that  $(\mathbb{R}, T)$  is connected.
14. a) Define a simple chain in a topological space  $(X, T)$  and a covering of  $X$ . Let  $(X, T)$  be a connected space, let  $O$  be an open cover of  $X$  and let  $a, b$  be distinct points of  $X$ . Prove that there is a simple chain consisting of members of  $O$  that connects  $a$  and  $b$ .
- b) Define a pathwise connected space and show that the topologist's sine curve is not pathwise connected.
15. a) Define a locally connected space. Prove that a topological space is locally connected if and only if each component of each open set is open.
- b) When is a topological space said to be
- Totally disconnected
  - 0-dimensional
  - a  $T_0$ -Space?
- c) Prove that every 0-dimensional  $T_0$ -Space is totally disconnected.



Reg. No. : .....

Name : .....

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MATHEMATICS

MAT1C04 : BASIC TOPOLOGY



Time : 3 Hours

Max. Marks : 80

**Instructions:**Answer any **Four** questions from Part-A. Each question carries **4** marksAnswer any **Four** questions from Part-B without omitting any unit. Each question carries **16** marks**PART-A**

- Give an example of a set  $X$  and topologies  $T_1$  and  $T_2$  on  $X$  such that  $T_1 \cup T_2$  is not a topology on  $X$ .
- Consider  $\mathbb{R}$  with the usual metric and  $\mathbb{Q}$  with the subspace metric. Is,  $\mathbb{Q}$  of the first category? Why?
- Prove that the first countability axiom is a hereditary property.
- Let  $\nu$  be the usual topology on  $\mathbb{R}$ . Describe the weak topology on  $\mathbb{R}$  induced by the function  $i: \mathbb{R} \rightarrow (\mathbb{R}, \nu)$  defined by  $i(x) = x$ .
- Prove that the closed interval  $[0, 1]$  has the fixed point property.
- Prove that continuous image of a pathwise connected space is Path wise connected.

**PART-B****Unit-I**

- a) Define a subbasis for a topology on a set  $X$ . Illustrate with an example.





- b) Let  $X$  be a set and  $\zeta$  be a collection of subsets of  $X$  such that  $X = \cup\{s : s \in \zeta\}$ . Prove that there is a unique topology  $T$  on  $X$  such that  $\zeta$  is a subbasis for  $T$ .
- c) Define
- First countable space
  - Second countable space
  - Prove that every second countable space is first countable. Is the converse true? Justify your answer
8. a) Let  $A$  be a subset of a topological space  $(X, T)$  and let  $x \in X$ . Prove that  $x \in \bar{A}$  if and only if every neighborhood of  $x$  has a non empty intersection with  $A$ . Also prove that  $\bar{\bar{A}} = \bar{A} \cup A'$ .
- b) Let  $A, B$  be subsets of a topological space  $(X, T)$  prove that  $\text{int}(A) \cup \text{int}(B) \subseteq \text{int}(A \cup B)$  and show by an example that equality need not hold.
- c) Let  $X = \{1, 2, 3, 4, 5\}$  and  $T = \{\emptyset, \{1\}, \{3, 4\}, \{1, 3, 4\}, \{2, 3, 4, 5\}, X\}$ . Find the closed subsets of with respect to  $T$ .
9. a) Prove that every convergent sequence in a metric space is a Cauchy sequence.
- b) Let  $(X, d)$  be a complete metric space and let  $A$  be subset of  $X$  with the subspace metric  $\rho$ . Prove that  $(A, \rho)$  is complete if and only if  $A$  is a closed subset of  $X$ .
- c) Let  $(x, T)$  and  $(y, \nu)$  be topological spaces. Define a continuous function  $f: X \rightarrow Y$ . If  $(X, T)$  is first countable and for each  $x \in X$  and each sequence  $\langle x_n \rangle$  in  $x$  such that  $x_n \rightarrow x$ , the sequence  $\langle f(x_n) \rangle$  converges to  $f(x)$ , then prove that  $f$  is continuous.



## Unit-II

10. a) Let  $(A, T_A)$  be a subspace of a topological space  $(X, T)$ . Prove that a subset  $C$  of  $A$  is closed in  $(A, T_A)$  if and only if there is a closed subset  $D$  of  $(X, T)$  such that  $C = A \cap D$ .
- b) Prove or disprove : Separability is a hereditary property.
- c) Define an embedding one topological space in another topological space and show that  $\mathbb{R}$  with usual topology can be embedded in  $\mathbb{R}^2$  with the usual topology
11. a) Define the product space of two topological spaces  $(X, T_1)$  and  $(Y, T_2)$ . Let  $(X_1, d_1)$  and  $(X_2, d_2)$  be metric spaces, for each  $i = 1, 2$ , Let  $T_i$  be the topology on  $X_i$  generated by  $d_i$ . Prove that the product topology on  $X = X_1 \times X_2$  is same as the topology on  $X$  generated by the product metric.
- b) Let  $(X, T), (Y_1, \nu_1)$  and  $(Y_2, \nu_2)$  be topological spaces and let  $f_1: X \rightarrow Y_1$  and  $f_2: X \rightarrow Y_2$  be functions, and define  $f: X \rightarrow Y_1 \times Y_2$  by  $f(x) = (f_1(x), f_2(x))$ . Prove that  $f$  is continuous if and only if  $f_1$  and  $f_2$  are continuous.
12. a) Let  $\{(X_\alpha, T_\alpha) : \alpha \in \Lambda\}^2$  be an indexed family of topological spaces and for each  $\alpha$  in  $\Lambda$  let  $(A_\alpha, T_{A_\alpha})$  be a subspace of  $(X_\alpha, T_\alpha)$ . Prove that the product topology on  $\prod_{\alpha \in \Lambda} A_\alpha$  is same as the subspace topology on  $\prod_{\alpha \in \Lambda} A_\alpha$  determined by the product topology on  $\prod_{\alpha \in \Lambda} X_\alpha$ .
- b) Let  $\{(X_\alpha, T_\alpha) : \alpha \in \Lambda\}$  be a family of topological spaces and let  $X = \prod_{\alpha \in \Lambda} X_\alpha$ . Prove that the product space  $(X, T)$  is second countable if and only if  $(X_\alpha, T_\alpha)$  is second countable for all  $\alpha \in \Lambda$  and  $T_\alpha$  is the trivial topology for all but a countable number of  $\alpha$ .